



Topics in consumption and production theory

Course leader: Ugo Gragnolati

Other instructors: –

Aims of the course

The course aims to introduce graduate students in economics to standard microeconomic models of consumption and production, and to the related mathematical formalism and deductive methods. Specifically, consumption and production choices at the individual level are presented with an axiomatic approach emphasizing theoretical primitives and key assumptions, from which subsequent implications are then formally derived. On the side of consumer choices, such an approach unfolds from the preference relation to utility maximization and expenditure minimization, which ultimately lead to derive and characterize the demand correspondence and the expenditure function. On the side of production choices, such an approach unfolds from technology to profit maximization and cost minimization, which ultimately lead to derive and characterize the supply correspondence and the cost function.

Learning outcomes and competences

Upon completion of the course, students will be familiar with some advanced theoretical concepts underpinning demand and supply analysis. Moreover, they will be able to define and critically evaluate the key assumptions underlying such analysis. Finally, students will develop mathematical competences for evaluating some of the most typical optimization problems in economics, particularly by characterizing their set of solutions starting from the properties of the objective function and of the underlying feasible set.



Pre-requisites

The course draws on basic notions in set theory, multivariate calculus, and constrained optimization. Students should be fully proficient in using and computing derivatives, while at least being acquainted with the Lagrangian method. At any rate, some basic mathematical notions and tools will be shortly recalled in class, as well as in a separate mathematical appendix included in the course materials.

Course contents and syllabus

The course is divided into two parts, dealing respectively with consumption and production theory. These two parts follow a common structure, as they define a key optimization problem and its dual, to then assess the properties of the set of solutions and value functions. The broad list of topics associated with the course is:

1. Consumption.
 - 1.1. *Consumption set and budget set.* The properties of the consumption and budget set are presented, particularly by introducing the notions of closedness, boundedness, compactness, and convexity of a set.
 - 1.2. *Preference relation.* The properties of the preference relation are presented, particularly by discussing completeness, transitivity, continuity, monotonicity/local non satiation, and convexity.
 - 1.3. *Utility function.* The utility function is introduced through Debreu's utility representation theorem. Special emphasis is given to the link between the convexity of preferences and the quasi-concavity of the utility function.
 - 1.4. *Utility maximization problem.* The consumer problem is formally introduced as a constrained maximization problem, thus defining the Marshallian demand correspondence and the indirect utility function. Special emphasis is given to characterizing the link between the assumptions on the preference relation and the resulting properties of the utility maximization problem. The Lagrangian method is introduced, focusing on first-order conditions of the utility maximization problem and on the economic interpretation of the Lagrange multiplier.



1.5. *Comparative statics*. Stability of the Marshallian demand correspondence for underlying parameter variations is discussed through Berge's maximum theorem. The income-consumption path and the Engel curve are defined. The effects of price variations on Marshallian demand are discussed by defining the substitution effect and its relationship with revealed preferences, the income effect, Slutsky compensation and decomposition, and the properties of the Slutsky matrix.

1.6. *Expenditure minimization problem*. The dual of the utility maximization problem is defined, thus introducing the Hicksian demand correspondence and the expenditure function. Hicksian demand is linked to the expenditure function via the envelope theorem and Shephard's lemma. The concavity of the expenditure function is discussed, linking it to the negative semidefiniteness of the Slutsky matrix under differentiability and regularity conditions.

2. Production.

2.1. *Technology and the production set*. The transformation function is introduced as a way to define the production set. The properties of the production set are presented, namely non-emptiness, closedness, irreversibility, free disposal, inaction, no free lunch, and returns to scale.

2.2. *Production function*. The case of single-output technology is introduced, emphasizing the link between the convexity of the production set and the concavity of the production function.

2.3. *Profit maximization problem*. The firm's problem is formally introduced as a constrained maximization problem, thus defining the supply correspondence and the profit function. First-order conditions of the profit maximization problem are presented and discussed. Previous results from consumer theory are used to discuss some of the properties of the supply correspondence and profit function. Further attention is specifically devoted to the convexity of the profit function in prices and to the positive semidefiniteness of its Hessian under differentiability and regularity conditions.

2.4. *Cost minimization problem*. The dual of the profit maximization problem is introduced, thus defining the conditional factor demand correspondence and the



cost function. The cost minimization problem is addressed with the Lagrangian method, focusing on first-order conditions and on the economic interpretation of the Lagrange multiplier. The properties of conditional factor demand correspondences and of the cost function are presented, tracing a parallel with the corresponding properties of the supply correspondence and profit function. Short-run and long-run cost functions are briefly compared.

Given **time constraints**, coverage will prioritize core items, so that some topics may be treated more briefly or omitted. Relatedly, some formal proofs may be skipped during lectures.

Organization of the course

The course consists of 12 hours of lectures. The relevant materials can be found in the *Teams class* “PhD Programme in Economics and Business” following this path: Documenti/General/Class Materials/ConsumptionProduction. First year PhD students are made members by using their UniCa email account. Students can access the *Teams* application by using the same account. The timetable of the course can be found in the Team class calendar.

Assessment method

The course does not contemplate an exam. If they wish, students can self-assess their level of understanding through an individual meeting with the instructor.

Reading list

The main reference for the course is:

- Mas-Colell, A., Whinston, M.D., and Green, J.R. *Microeconomic Theory*. Oxford University Press, 1995.

The course focuses on selected topics in **chapters 1, 2, 3, and 5** of the above textbook.

Other information

Students can contact the instructor by email to fix office hours if needed.