MATHEMATICAL ASPECTS OF GAME THEORY

ANTONIO IANNIZZOTTO

The aim of this course is to introduce graduate or PhD. students to the basic features of classical game theory, from a strictly mathematical point of view. Knowledge of topology and a bit of functional analysis are prerequisite.

1. **Introduction: games, solutions, and equilibria.** Definitions of game, strategy, choice, utility; representation theorem; classification of games; games with two players and payoff matrix; dominations; solution of a game by iterated elimination of strategies.
4. **Some set-valued analysis.** Definitions of set-valued mappings, domain, graph, inverse mapping; types of continuity; mappings with compact, connected values, closed or compact graph; Michael's selection theorem; fixed point theorems of Kakutani, Sion, Browder, and Nadler; KKM principle.
5. **Mixed strategies and Nash equilibria.** Definition of Nash equilibrium; mixed strategies; Pareto optimum; approximated equilibria.
6. **Zero sum games and the minimax problem.** Zero sum games; saddle points; minimax theorems of Von Neumann, Fan-Sion, König, Ricceri.
7. **Cooperative games.** Definitions of deal, kernel, solution; Stapley value.
8. **Dynamic games.** Definition of dynamic game; adaptation of strategies; credibility and probability; bayesian solution; representation through graphs; sub-games; Selten's theorem.
9. **Examples and applications.** Odds & evens; rock, paper & scissors; battle of the sexes; production game; Cournot's and Stackelberg's duopoly models; entry game; bank run; liberal dilemma; prisoner's dilemma.

References

[1] J.P. Aubin, Mathematical methods of game and economic theory, North-Holland, Amsterdam (1979).
[2] J.P. Aubin, H. Frankowska, Set-valued analysis, Birkhäuser, Boston (1990).
[3] O. Morgenstern, J. von Neumann, Theory of games and economic behavior, Princeton University Press, Princeton (1953).
[4] J. Nash, Non-cooperative games, *Ann. Math.* **54** (1951) 286–295.
[5] B. Ricceri, S. Simons, Minimax theory and applications, Kluwer, Dordrecht (1998).
[6] J. von Neumann, Zur Theorie der Gesellschaftsspiele, *Math. Ann.* **100** (1928) 295–320.

Dipartimento di Matematica e Informatica
Università degli Studi di Cagliari
Viale L. Merello 92, 09123 Cagliari, Italy
E-mail address: antonio.iannizzotto@unica.it