Timing Analysis of Accreting Millisecond X-ray Pulsars

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ACRONYMS

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<tr>
<td>dof</td>
<td>degree of freedom</td>
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<tr>
<td>LMXB</td>
<td>Low Mass X-ray Binary</td>
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<tr>
<td>NS</td>
<td>Neutron Star</td>
</tr>
<tr>
<td>PCA</td>
<td>Proportional Counter Array</td>
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<tr>
<td>PCU</td>
<td>Proportional Counter Unit</td>
</tr>
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<td>QPO</td>
<td>Quasi Periodical Oscillation</td>
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<td>RXTE</td>
<td>Rossi X-Ray Timing Explorer</td>
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INTRODUCTION

Scope of this thesis is to present the work done during the three years of my PhD. The main object of my research activity was the study of the temporal evolution of a sub-class of Low Mass X-ray Binaries: the Accreting Millisecond X-Ray Pulsars, and in particular the study of the temporal evolution of the spin frequency. In the following a brief description of such systems is given, pointing out the peculiar physical conditions in which these systems are and why is worth their study.

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1.1 X-RAY BINARIES

The brightest X-ray galactic sources are interacting binaries in which one of the two star is a Neutron Star (NS) or a Black Hole (BH). Such system are so tight that matter can pass from the outer layer of the companion star onto the compact object. The process just described is called accretion. The characteristic X-ray emission comes from the release of gravitational potential energy by the accreting matter flowing in the disk and/or impacting the object surface (if exist). The more compact the object is, the more energetic the accretion process is. The origin of the compact object is usually ascribed to a Supernova explosion, although other evolutionary channels are possible, like the accretion induced collapse of a White Dwarf [Verbunt, 1993]. X-ray binaries shows several temporal variability, both in intensity and spectral shape and on long and short timescales. Since we do not studied spectral variabilities in our work, we will not describe them. One of the most evident time variabilities is that, although some X-ray binaries are persistent sources, that is their fluxes not vary significantly, the most are transient sources, that is they have low X-ray luminosity (in quiescence $L \lesssim 10^{33}$ erg sec$^{-1}$) for most of the time and become very bright ($L \gtrsim 10^{36}$ erg sec$^{-1}$) rapidly and then decay to quiescence. In most cases the mechanism which switch on and off a transient source is still not understood.

According to the mass of the companion star, X-ray binaries are divided in Low Mass X-ray Binaries (LMXBs) and High Mass X-ray Bi-
2 Introduction

Figure 1: Artistic impression of a X-ray binary system. In particular the system represented is a LMXB where the companion star has filled its Roche lobe. The image was produced with the program BinSim developed by Rob Hynes. The peculiar components of a X-ray Binary are labelled.

naries (HMXBs)[Bradt e McClintock, 1983]. In LMXBs the companion mass is lower than $1M_\odot$, typically a Main sequence star, a Brown dwarf, a White Dwarf or a He core (slightly evolved) star. In the HMXBs the companion star is a $\gtrsim 5M_\odot$ main sequence giant OB star. This classification, simply based on the companion mass, put in evidence several other important differences like age of the binary system, evolutionary scenario, accretion modes and magnetic field strength between LMXBs and HMXBs.

1.1.1 High Mass X-Ray Binaries

High Mass X-Ray Binaries (HMXBs), due to the presence of a main sequence massive star, are young systems (the lifetime of a giant OB star in the main sequence is $\sim 10^7$ years), and, when present, the NS in these systems usually has a strong magnetic field ($\sim 10^{12}$ G). The companion star emits a strong stellar wind and the accretion can occur via both stellar wind and accretion disk. The presence of a coherent flux modulation at the spin period of the NS, with periods in most cases between 0.1 and 1000 seconds, is common and is a patent evidence that the compact object is a NS and not a BH.
1.1.2 Low Mass X-Ray Binaries

As already pointed out, Low Mass X-ray Binaries (LMXBs) are binary systems in which one of the two objects is a neutron star or a black hole, and the other object, usually called companion star, is a low mass (< 1M_⊙) main sequence star or brown dwarf or a white dwarf. In case the compact object is a NS, typical values for the surface magnetic field are $10^8 - 10^9$ G. Such systems are so tight that the matter transfer is due to Roche lobe overflow. The matter, in order to reach the NS surface (or the last stable orbit, in case of BH), has to lose its angular momentum. To do this, the matter will form an accretion disk around the compact object. A large fraction of LMXBs are transient. Due to low magnetic field strength, pulsation are rarely observed in LMXBs. A phenomena occurring only in LMXBs are the Type-I bursts, which are due to ignition of nuclear fusion processes of the accreted matter on the NS surface under degenerate conditions. Such processes are extremely rapid: the X-ray flux became hundred times more intense in a fraction of seconds, reaching the Eddington luminosity, to rapidly (roughly) exponentially decay with time scales of the order of few tens of second or less.

1.1.3 Accretion Disks

The matter accreting from the companion star onto the compact object brings an high specific angular momentum. Due to the smallness of compact object (~ 10 km), the direct falling onto it is in practice impossible. The matter will then start to orbit around the compact object in Keplerian orbits, forming an accretion disk. Although the matter is in the state of plasma and thus a full magnetohydrodynamical treatment should be applied to describe accretion disks are among the few astronomical structures for which an analytical solution is known under some simplifying assumptions. This solution was given by Shakura e Syunyaev [1973]. The disk solution proposed by Shakura e Syunyaev [1973] is purely hydrodynamical, although electrodynamical effects due to the plasma nature of the matter are taken into account in the so-called “$\alpha$-prescription” for the viscosity. Such prescription permit an outward flow of angular momentum without which accretion onto the compact object is not possible. A straightforward application to the cases of disk around a BH is possible, but when the accreting object is a magnetised NS the disk dynamics is supposed to be strongly affected, at least very near to the NS surface, by its interaction with the NS magnetic field. Unfortunately, due to a very high complexity of the problem, an analytical solution for an accretion disk in presence of a strong magnetic field is still missing. Several attempts was done to found a solution, both theoretical [see Bozzo et al. 2008 for a review of Ghosh et al., 1977; Ghosh e Lamb, 1979a,b; Wang, 1987, 1995; Rappaport et al., 2004] and numerical [see Romanova et al., 2002, for a very complete review up to 2002], in order to find the answer to several open questions, like [citing Romanova et al., 2002]: “... (1) What is the structure of the disk near the magnetized star? (2) Where is the inner radius of the disk? (3) What is the nature of the
funnel flows? For example, which force is dominant in lifting matter to the funnel flow? (4) How is the accretion rate influenced by the star’s magnetic moment and angular velocity? (5) What is the mechanism of angular momentum transport between the star and the disk? What determines whether the star spins up or spins down? and (6) What are the necessary conditions for magneto-centrifugally driven outflows from the disk and/or the star?”. The study of the timing behaviour of the spin frequency of the accreting millisecond X-ray pulsars is of fundamental importance to give constraints and clues of the interaction between the NS magnetosphere and the accreting matter in what is called the “threaded disk model”.

1.1.4 Accreting Millisecond X-Ray Pulsars

The Accreting Millisecond X-ray Pulsars (AMXPs) are a sub-class of the LMXBs. The only observational difference between LMXBs and AMXPs is, up to now, the presence of a coherent flux modulation with periods of the order of 1 ms. The coherent modulation of the flux is due to the matter accreting onto the polar caps, which create hot spots. If the magnetic and rotational axes are misaligned, the flux from the hot spot is modulated by the rotation in a lighthouse effect. Due to the huge moment of inertia of a NS ($10^{45}$ gr cm$^2$) the rotational frequency is extremely stable. The magnetic field must be strong enough ($\gtrsim 10^8$ G) in order to drive the accreting matter onto the polar caps. The accreting matter has a specific angular momentum greater than the NS one, since it rotates in the disk in Keplerian orbits. Then it exert a positive torque onto the NS, spinning it up. But there are some experimental evidences that an AMXPs can spin-down during the accretion [see, e.g. Papitto et al., 2007; Burderi et al., 2006]. As mentioned above, spin-down during accretion can be explained by disk threading effects [see, e.g. Rappaport et al., 2004], in which the magnetic field allows to exchange angular momentum from the NS to the accretion disk, with a negative torque onto the NS, in such a way to overwhelm the positive material torque.

Up to now only 10 of such sources are known (see Table 1). All the AMXPs are transient sources, and only for three of them was observed more than one outburst: SAX J1808.4–3658, Aql X–1 and SAX J1748.9–2021. In the last two the pulsation was discovered recently [Casella et al., 2007; Altamirano et al., 2008] analysing archival data. SAX J1808.4–3658 is not only the first ever AMXP ever discovered, but is the only one for which five outburst were observed and in each one the pulsation was clearly visible during the outburst. This huge dataset spanning ten years has allowed us to measure with great accuracy the orbital period derivative (see Chapters 4 and 5).

1.2 Instrumentation and Techniques

Since the Earth atmosphere is (mercifully) opaque to photons in the ultra-violet and X-ray bands, the sky observation in these energy band is possible only from satellites. The first “picture” of the X-ray sky, signing the begin of the X-ray astronomy, was performed on 18 June 1962 by the Geiger counters on board of a Aerobee missile in sub-
Table 1: The 10 known AMXPs

<table>
<thead>
<tr>
<th>Source Name</th>
<th>$P_{\text{spin}}$ (ms)</th>
<th>$P_{\text{orb}}$</th>
<th>Pulsation discovery</th>
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<tbody>
<tr>
<td>IGR J00291+5934</td>
<td>1.7</td>
<td>2.5hr</td>
<td>Galloway et al. [2005]</td>
</tr>
<tr>
<td>Aql X-1</td>
<td>1.8</td>
<td>19hr(*)</td>
<td>Casella et al. [2007]</td>
</tr>
<tr>
<td>SAX J1748.9-2021</td>
<td>2.3</td>
<td>8.8hr</td>
<td>Altamirano et al. [2008]</td>
</tr>
<tr>
<td>XTE J1751-305</td>
<td>2.3</td>
<td>42m</td>
<td>Markwardt e Swank [2002]</td>
</tr>
<tr>
<td>SAX J1808.4-3658</td>
<td>2.5</td>
<td>2hr</td>
<td>Wijnands e van der Klis [1998]</td>
</tr>
<tr>
<td>HETE J1900.1-2455</td>
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<td>1.4hr</td>
<td>Morgan et al. [2005]</td>
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<td>XTE J1807-294</td>
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<td>40m</td>
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<td>43.6m</td>
<td>Galloway et al. [2002]</td>
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<tr>
<td>SWIFT J1756.9-2508</td>
<td>5.5</td>
<td>54m</td>
<td>Krimm et al. [2007]</td>
</tr>
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orbital trajectory by Giacconi et al. [1962]. Although the angular resolution of this first measurement was very poor, these measurements serendipitously give evidence of extra-solar X-ray sources, that was later identified as the X-ray binary system Sco X-1. Since then a lot of satellites were launched to observe the X-ray sky.

1.2.1 The Rossi X-ray Timing Explorer

We will briefly describe the most important instrument for our work: the Rossi X-ray Timing Explorer [RXTE Bradt et al., 1993], launched on 30 December 1995. Up to very recently it was the only one with time capabilities and effective area necessary to study in detail the temporal behaviour of the X-ray sources. RXTE payload is composed by three instruments: the Proportional Counters Array [PCA Zhang et al., 1993; Jahoda et al., 2006], the High Energy X-ray Timing Experiment (HEXTE) and the All Sky Monitor (ASM). The ASM, based on coded masks, observes ~ 80% of the sky each orbit (~ 90 min) with a spatial resolution of 3’ x 15’, it operates in the 1.5-12 keV range and has a time resolution of 1/8 seconds. The ASM was conceived with the scope to rapidly spot transient sources and state transition in the X-ray sources, triggering the other instruments and the scientific community to observe the phenomena in the shortest time possible.

The HEXTE has a field of view of ~ 1° and operates in the 15-200 keV range. It consists of two photon counter detectors, each having an area of ~ 800 cm². We never used data from this instrument.

The PCA constitute the principal instrument on board RXTE. It is composed by an array of 5 collimated Proportional Counter Units (PCUs), having the same field of view (~ 1°) of HEXTE. The total geometrical area is near 1 m², while the effective area is of ~ 6000 cm². The energy range in which operates is 2-60 keV and the photon arrival time resolution is 1µs, although, in order to save telemetry, data are usually packed with a resolution of 125 µs.
Unfortunately, although still operating, RXTE is old and shows the signs of aging. In fact, of the five PCUs, only two still work properly. Nevertheless it will be maintained on duty for another year.

1.2.2 The XMM-Newton

Very recently we performed the timing analysis (see Chapter 5) using data from the European Space Agency’s (ESA) X-ray Multi-Mirror Mission (XMM-Newton). XMM-Newton was launched on 10 December 1999. It is composed by 3 X-ray telescopes, each one consisting of 58 Wolter I grazing-incidence mirrors which are nested in a coaxial and co-focal configuration, and an optical monitor, the first flown on a X-ray observatory. The large collecting area and ability to make long uninterrupted exposures because it is launched in a very elongated (eccentric) orbit with an orbital period of 48 hours, provide highly sensitive observations. The field of view of XMM-Newton is ~ 30′ with good spectral resolution using the European Photon Imaging Camera (EPIC), which consists of one pn CCD and two MOS CCD arrays. High-resolution spectroscopy capabilities are provided by the Reflection Grating Spectrometer (RGS) that deflects half of the beam of two of the X-ray telescopes (those with the MOS CCDs). More important for us is the EPIC pn which offers very high time resolution (~ 30µs).

1.3 OUTLINE

A brief description of the arguments discussed in this thesis, chapter by chapter, is given.

IN THE SECOND CHAPTER the timing analysis of the source XTE J1807–294 is presented. A particular timing technique developed by us is described and applied to XTE J1807–294 in order to derive for
Figure 3: Diagram of the XMM-Newton satellite.

the first time a complete orbital solution. Such technique permit us to virtually eliminate the strong pulse phase delays erratic behaviour showed by this source, leaving unaltered the contributes due to the orbital residuals.

IN THE THIRD CHAPTER, on the basis of the work presented in Chapter 2, we studied the spin evolution of XTE J1807–294 giving for the first time a measure of the spin-up of the source and an estimate of the distance. A detailed discussion above the source position error propagation in that measure is given.

IN THE FORTH CHAPTER a timing analysis of the four outbursts of the source SAX J1808.4–3658 is given. We reported, for the first time, the orbital period derivative, along with an improved set of orbital parameters. Such measures permit us to constrain the nature of the companion star. Moreover we are forced to concluded that nearly all the matter lost by the companion star is lost from the inner Lagrangian point and not accreted onto the NS.

THE FIFTH CHAPTER, although very similar to the previous one, present the first timing analysis of an AMXP performed on XMM-Newton data of the very recently outburst of SAX J1808.4–3658. The simultaneous timing analysis of five outburst, spanning a time interval of ten years, permit to confirm the results described in Chapter 4, confirming the amazing evolutionary scenario.
2.1 Abstract

We describe a timing technique that allows to obtain precise orbital parameters of an accreting millisecond pulsar in those cases in which intrinsic variations of the phase delays (caused e.g. by proper variation of the spin frequency) with characteristic timescale longer than the orbital period do not allow to fit the orbital parameters over a long observation (tens of days). We show under which conditions this method can be applied and show the results obtained applying this method to the 2003 outburst observed by RXTE of the accreting millisecond pulsar XTE J1807–294 which shows in its phase delays a non-negligible erratic behavior. We refined the orbital parameters of XTE J1807–294 using all the 90 days in which the pulsation is visible and the method applicable. In this way we obtain the orbital parameters of the source with a precision more than one order of magnitude better than the previous available orbital solution, a precision obtained to date, on an accreting millisecond pulsar, only for SAX J1808.4–3658 analyzing three outbursts spanning over four years and with a much better statistics.

2.2 Introduction

Low Mass X-ray Binaries (LMXB) are binary systems in which one of the two stars is a neutron star (NS) with low magnetic field ($< 10^9$ Gauss) which accretes matter from a low-mass ($< 1 M_\odot$) companion star. According to the so-called recycling scenario [see for a review Bhattacharya e van den Heuvel, 1991] millisecond radio pulsars originate from LMXBs, where the accretion torques and the relatively weak magnetic fields are able to spin up the NSs up to millisecond periods. When the companion star stops transferring matter to the NS, the NS can switch on as millisecond radio pulsar.

A striking confirmation of this scenario was the discovery in 1998 of millisecond X-ray pulsars in transient LMXBs. The first LMXB observed to show coherent pulsations at a frequency of $\sim 400$ Hz was
the well studied SAX J1808.4–3658 [Wijnands e van der Klis, 1998; Chakrabarty e Morgan, 1998]. Due to the low magnetic fields of these sources, the chance to see a pulsed emission from a LMXB is quite low. However, to date 7 LMXBs were discovered to be accreting millisecond pulsars [Wijnands, 2005], and all of them are in transient systems. They spend most of the time in a quiescent state, with very low luminosities (of the order of $10^{31} - 10^{32}$ ergs/s) and rarely we go into an X-ray outbursts with luminosities in the range $10^{36} - 10^{37}$ ergs/s. Indeed, of all these sources only SAX J1808.4–3658, which shows more or less regular outbursts every two years, has been observed in outburst more than once with RXTE. All the other sources have shown just one outburst in the RXTE era.

This fact makes the study of the timing properties of these sources even more difficult, given that the duration of the observations is not a matter of choice, but is conditioned by the duration of the outbursts which, in turns, puts a constrain on the precision of the parameters that we can derive. And this is also the reason why we have to obtain all the information and the precision of the parameters we need just using the available data. In the case of accreting millisecond pulsars, among the parameters of interest there are, of course, the timing parameters, that are the orbital parameters and the spin parameters. The orbital parameters can give us important information on the binary system, its evolution, and even on the nature (e.g. degenerate or not) of the companion star. Also, a precise orbital solution will be important for a precise determination of the spin parameters, the spin period evolution and the accretion torques acting onto the NS.

As already mentioned above, the knowledge with the maximum possible precision of the orbital parameters is of fundamental importance in itself and for a successive study of the spin and the spin variations. The study of the frequency Doppler shift due to orbital motion of a millisecond pulsar in a binary system is the first step to obtain an estimate of the set of orbital parameters. To refine this estimate the next step is the study of the pulse phase shifts in order to obtain differential corrections to the orbital parameters and therefore a finer orbital solution. However, in some cases, not all the data in which the coherent X-ray pulsations are visible can be used to obtain the differential corrections. The pulse phase shifts are frequently affected by intrinsic long-term variations and/or fluctuations (probably caused by the accretion torques) which are superimposed to the modulation due to the orbital motion of the source. Clear examples of these complex behaviors of the pulse phase shifts in accreting millisecond pulsars can be found in Burderi et al. [2006], who analyze SAX J1808.4–3658 and find a big jump in the pulse phase shifts of the fundamental harmonic of the pulse, and in Papitto et al. [2007], who analyze XTE J1814–338 finding a modulation of the pulse phase shifts, anti-correlated to the X-ray flux, superposed on a general spin-down trend.

Of course, the presence of non negligible variations of the pulse phase shifts with time, make it very difficult to fit a long dataset with a sinusoid in order to obtain a precise estimate of the orbital parameters using all the available time-span. It is then necessary, in order to decouple the orbital modulation from the proper fluctuations of the pulse phases, to temporarily eliminate the latter in some way. This is
often impossible to obtain by a fit with a simple model, due to the observed complex behaviors and/or our poor knowledge of the physics of the accretion torques. In those cases we are forced to restrict the fit differential corrections of the orbital parameters on short time intervals, in which the intrinsic variations of the phase shifts can be safely approximated with simple model, e.g. a parabola, or even neglected.

In this paper we describe a simple method which permits, under some conditions, to remove from the pulse phase shifts all the effects not due to differential orbital parameters corrections. We apply this method to the source XTE J1807−294, obtaining for the first time a complete set of orbital parameters with a precision at least one order of magnitude higher with respect to the previously available orbital solution.

2.3 Observations

The millisecond X-ray pulsar XTE J1807−294 was spotted by RXTE on February 21\textsuperscript{st}, 2003 [Markwardt et al., 2003c]. The source was observed with PCA (Proportional Counter Array) and HXETE (High Energy X-ray Timing Experiment), the principal instruments on-board of RXTE, from February 28 to June 22, 2003. XTE J1807−294 was also observed with other satellites such as XMM-Newton [Campana et al., 2003; Kirsch et al., 2004; Falanga et al., 2005b], Chandra [Markwardt et al., 2003a] and Integral [Campana et al., 2003]. No optical or radio counterpart was reported. Linares et al. [2005] have reported the presence of twin kHz QPOs analyzing RXTE observation.

In literature several attempts have been done in order to derive the orbital parameters. Markwardt et al. [2003c] give only source position and the orbital period. Kirsch e Kendziorra [2003], analyzing the XMM-Newton observation of the outburst, give an estimate of the semi-major axis. The first complete set of orbital parameters was reported by Campana et al. [2003] analyzing the XMM-Newton observation of the outburst. Successively Kirsch et al. [2004] give a complete set of orbital parameters. More recently Falanga et al. [2005b], analyzing the XMM-Newton observation but with a more simple approach give a new set of orbital parameters. All these authors assumed as orbital period the period reported by Markwardt et al. [2003c].

Here we analyze all the archival RXTE observations of this source. In particular, we use data from the PCA (proportional counter array) instrument on board of the satellite RXTE. We use data collected in GoodXenon packing mode, with maximum time and energy resolution (respectively 1\textmu s and 256 energy channels). In order to improve the signal to noise ratio we select photon events from PCUs top layer and in the energy range 3-13 keV. Using the \textit{faxbary} tool (DE-405 solar system ephemeris) we corrected the photon arrival times for the motion of the earth-spacecraft system and reported them to barycentric dynamical times at the Solar System barycenter. We use the source position reported by Markwardt et al. [2003a] using the Chandra observation of the same outburst.
In order to test the goodness of the available orbital solution, we correct the photon arrival times with the formula:

\[ t_{em} \simeq t_{arr} - A \left[ \sin \left( m(t_{arr}) + \omega \right) + \frac{e}{2} \sin \left( 2m(t_{arr}) + \omega \right) - \frac{3e}{2} \sin \left( \omega \right) \right], \tag{2.1} \]

where \( t_{em} \) is the photon emission time, \( t_{arr} \) is the photon arrival time, \( A \) the projected semi-major axis in light seconds, \( m(t_{arr}) = 2\pi(t_{arr} - T^*)/P_{orb} \) is the mean anomaly, \( P_{orb} \) the orbital period, \( T^* \) is the time of ascending node passage, \( \omega \) is the periastron angle and \( e \) the eccentricity. We used the orbital parameters reported by Kirsch et al. [2004], adopting an eccentricity \( e = 0 \) (see Tab. 2 for details).

We divided the whole observation in time intervals of \( 1/6 P_{orb} \) length each and epoch-folded each of these data intervals with respect to the spin period we reported in Tab. 2. The pulse phase delays are obtained fitting each pulse profile with two sinusoidal components, since higher-order harmonics were detectable in the folded light curve. We fixed the period of the sinusoids to 1 and 0.5 times the spin period, respectively, and we used the phase of the fundamental harmonic to infer the pulse phase shifts. In Fig. 4 we show the pulse phase delays obtained in this way, where we have plotted only the pulse phase delays corresponding to the folded light curves for which the statistical significance for the presence of the X-ray pulsation is \( > 3\sigma \).

On one hand in Fig. 4 an orbital modulation it is clearly visible, on the other hand the erratic behavior is apparent. It can be seen that this behavior has characteristic time scales of the order of several \( P_{orb}^2 \).

### 2.4 Differential Corrections of the Orbital Parameters

We propose here a simple method of analysis which allows to eliminate, or at least strongly reduce, the long-term variation and erratic behavior of the pulse phase shifts. In particular, instead of plotting the obtained pulse phases as a function of time, we consider the phase difference between each interval and the following one, \( \Delta \phi(t_i) = \phi(t_{i+1}) - \phi(t_i) \), in the hypothesis that for each \( i \) we have \( t_{i+1} - t_i = \Delta t \), where \( \Delta t \) is constant during all the observation. In this way we obtain the phase shifts shown in Fig. 6, where, as it is easy to see, the orbital modulation is still visible, but any long-term variation of the pulse phases is completely smoothed out. In this way we apply a linear filter to the pulse phase delays, for which we illustrate the fundamental properties. We now use the term input to indicate the original signal, that is the pulse phase delays vs. time, and output to indicate the signal we obtain plotting the phase difference of each interval with the following vs. time. When the input signal is a sinusoid of period \( P \), the output is another sinusoid with same period but with different phase and amplitude. In Fig. 5 we report the gain \( G \), that is the ratio of the amplitudes of the output to the input signal, for a sinusoidal signal of period \( P \). The analytical expression for \( G \) is: \( G = 2 \sin(\pi \Delta t/P) \). As can be seen in Fig. 5 \( G \) has the maximum for \( P = 2\Delta t \) coincident with the Nyquist frequency. For \( P >> \Delta t \) we have \( G \propto P^{-1} \). This filter is then a band-pass filter, limited at high frequencies by the Nyquist fre-
We can then analyze separately the response to the filter of the Doppler frequency and at low frequencies we can fix a limit at the period $P \approx 12\Delta t$, at which the amplitude is reduced by an order of magnitude. For period of $P \approx 60\Delta t$ the amplitude is reduced by a factor ten.

Due to its linearity the application of this filter to a signal which is the sum of several signals is equal to the sum of each filtered signal. We can then analyze separately the response to the filter of the Doppler shift due to the orbital motion without fear that the erratic behavior of the source can alter the result.

The residual in the phase delays due to a non-perfectly corrected orbital parameters is given by the expression:

$$
\phi_{\text{orb}}(t) = P_{\text{spin}}^{-1} \left( \sin (m(t) + \omega) + \frac{e}{2} \sin (2m(t) + \omega) - \frac{3e}{2} \sin (\omega) \right) \ dA - \\
\frac{2\pi A}{P_{\text{orb}}} \left( \cos (m(t) + \omega) + e \cos (2m(t) + \omega) \right) \ dT^* - \\
\frac{m(t)/A}{P_{\text{orb}}} \left( \cos (m(t) + \omega) + e \cos (2m(t) + \omega) \right) \ dP_{\text{orb}} + \\
A \left( \frac{1}{2} \sin (2m(t) + \omega) - \frac{3}{2} \sin (\omega) \right) \ de + \\
\left( A \cos (m(t) + \omega) + \frac{e}{2} \cos (2m(t) + \omega) - \frac{3e}{2} \cos (\omega) \right) \ d\omega \ (2.2)
$$

where $P_{\text{spin}}$ is the spin period with respect to the light curves are folded and $dA, dT^*, dP_{\text{orb}}, de$ and $d\omega$ are the differential correction of the orbital parameters (the projected semi-major axis, the time of ascending node passage, the periastron angle, and the eccentricity, respectively).

If we compute difference between the phase of two adjacent folded light curves we obtain, for $\Delta \phi_{\text{orb}}(t_i)$, the expression:

$$
\Delta \phi_{\text{orb}}(t_i) = \phi_{\text{orb}}(t_{i+1}) - \phi_{\text{orb}}(t_i) = P_{\text{spin}}^{-1} \left[ 2 \cos (m_i + \omega + m_\Delta/2) \sin (m_\Delta/2) + e \cos (2m_i + \omega + m_\Delta \sin (m_\Delta) ) \right] \ dA + \\
\frac{4\pi A}{P_{\text{orb}}} \left[ \sin (m_i + \omega + m_\Delta/2) \sin (m_\Delta/2) + e \sin (2m_i + \omega + m_\Delta \sin (m_\Delta) ) \right] \ dT^* + \\
\frac{2m_i/A}{P_{\text{orb}}} \left[ \sin (m_i + \omega + m_\Delta/2) \sin (m_\Delta/2) + e \sin (2m_i + \omega + m_\Delta \sin (m_\Delta) ) \right] - \\
\frac{m_\Delta A}{P_{\text{orb}}} \left[ \cos (m_i + m_\Delta + \omega) + e \cos (2m_i + 2m_\Delta + \omega) \right] \ dP_{\text{orb}} + \\
A \cos (2m_i + \omega + m_\Delta \sin (m_\Delta) ) \ de - \\
\left[ 2 \sin (m_i + \omega + m_\Delta/2) \sin (m_\Delta/2) + e \sin (2m_i + \omega + m_\Delta \sin (m_\Delta) ) \right] \ d\omega \ (2.3)
$$

where we pose for simplicity $m(t_i) = m_i$ and $2\pi\Delta t/P_{\text{orb}} = m_{i+1} - m_i = m_\Delta$.

2.5 DATA ANALYSIS

We have applied the technique described above to the PCA data of XTE J1807–294. In particular, we have used the phase delays of Fig.4 in order to calculate for each time interval the phase difference with respect to the following interval, and these are plotted vs. time in Fig.6.
We consider only phase differences between contiguous time intervals and exclude the phase differences between interval separated by gaps in time. The errors on the phase differences are propagated summing in quadrature the errors on the phases from which the difference is calculated, that is \( \sigma_{\Delta \phi(t_i)}^2 = \sigma_{\phi(t_{i+1})}^2 + \sigma_{\phi(t_i)}^2 \). From the figure it is apparent that the long term variation and the erratic behavior of the phase shifts is now completely smoothed out. We can therefore proceed to fit with Eq. 2.3 these phase difference over the whole period in which the coherent pulsation was detectable. In this way we obtain a very good fit of the data. To show the goodness of the fit we plot in Fig. 7 the phase differences between days 10 and 11 from the start of the outburst; the dashed line is the best fit sinusoidal modulation obtained from Eq. 2.3.

Eq. 2.3 is essentially a sum of sinusoidal terms with period equal to \( P_{\text{orb}} \) and \( P_{\text{orb}}/2 \). The latter are due only to the eccentricity. Then, to test if the orbit shows an eccentricity we epoch folded the light curves on a time interval \( \Delta t = P_{\text{orb}}/10 \) in order to have a sufficient number of points to sample each period to avoid aliasing phenomena. In fact, if we use, as before, intervals of length \( \Delta t = P_{\text{orb}}/6 \) this means that we sample the modulation at \( P_{\text{orb}}/2 \) (eventually due to a non negligible eccentricity) with only three points, and this can produce ambiguities in the results of the fit. Using instead time intervals of length \( \Delta t = P_{\text{orb}}/10 \), we sample the modulation with period \( P_{\text{orb}} \) with 10 points and the modulation with period \( P_{\text{orb}}/2 \) with 5 points, which is, as we have verified, a good compromise to get precise estimates of all the orbital parameters.

To check that the long-term trend visible in Fig. 4 has been indeed eliminated by the technique described above we describe the residual of the erratic behavior with a parabola, then we fit the phase differences with the expression:

\[
\Delta \phi(t) = a + b \, t + c \, t^2 + \Delta \phi_{\text{orb}}(t), \tag{2.4}
\]

where \( a, b \) and \( c \) are the coefficients of the parabola. There is no evidence of residuals due to the erratic behavior, and in the fit the \( a, b \) and \( c \) parameters result largely compatible with zero. Moreover the orbit does not show an appreciable eccentricity, \( e \), for which we find an upper limit (at 95% c.l.) of \( 3.6 \times 10^{-3} \). We also find that \( dT^* \) and \( d\omega \) result perfectly correlated, as expected for a circular orbit.

Due to these results we can safely make two assumptions: 1) the orbit is circular; 2) we can safely describe the residual simply with a constant. We epoch folded the light curves on a time interval \( \Delta t = P_{\text{orb}}/6 \) in order to have a better statistics. We then fit the phase difference with the simpler formula:

\[
\Delta \phi(t) = a + \Delta \phi_{\text{orb}}(t), \tag{2.5}
\]

where we fixed \( d e = d \omega = 0 \). We iterate this process until no residual are observed. In this way we find a good fit, corresponding to a \( \chi^2/d.o.f. \) of 1664.4/1142; the best fit parameters are reported in Tab. 2.

As can be seen in Fig. 8, our more precise orbital solution allows us to clearly detect the coherent pulsations up to 104 days since the start of the observation (June 12) with a detection confidence level of...
Table 2: Orbital Parameters for XTE J1807-294.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Other Works</th>
<th>This Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbital period, $P_{\text{orb}}$ (s)</td>
<td>2404.45(3)$^a$</td>
<td>2404.41665(40)</td>
</tr>
<tr>
<td>Projected semi-major axis, $a_x \sin i$ (lt-ms)</td>
<td>4.8(1)$^b$</td>
<td>4.819(4)</td>
</tr>
<tr>
<td>Ascending node passage, $T^*$ (MJD)</td>
<td>52720.67415(16)$^b$</td>
<td>52720.675603(6)</td>
</tr>
<tr>
<td>Eccentricity, $e$</td>
<td>-</td>
<td>$&lt; 0.0036$</td>
</tr>
<tr>
<td>Spin frequency, $\nu_0$ (Hz)</td>
<td>190.623508(15)$^b$</td>
<td>190.62350684(5)</td>
</tr>
</tbody>
</table>

Errors are intended to be at 1σ c.l., upper limits are given at 95% c.l.

$^a$Markwardt et al. [2003a].
$^b$Kirsch et al. [2004].

3.6σ, while between May 26 and June 10, although the source is still detectable, the pulsations are no more significantly detected.

2.6 CONCLUSIONS

We have described a simple technique that permits to drastically reduce the presence of erratic behavior and long-term intrinsic variation of the pulse phase delays of the source, thus allowing to fit the residual modulation of these phase shifts due to uncertainties in the orbital parameters of the system on a very long time-span and to obtain a precise measure of the orbital parameters. We applied this technique to the source XTE J1807–294, which shows the longest X-ray outburst observed by RXTE from an accreting millisecond pulsar. In this way we can fit the residual modulation of the phase differences over the whole time-span in which the coherent pulsations were significantly detected (about 104 days from the start of the outburst), obtaining a set of orbital parameters with a precision that is at least one order of magnitude better than the previously published orbital solutions for this source.
Figure 4: Plot of the pulse phase delays obtained epoch folding barycentered with respect the orbital parameters reported by Kirsch et al. [2004] on time intervals of \( P_{\text{orb}}/6 \). It is clearly visible an orbital modulation superimposed to the erratic behavior of the source.
Figure 5: Plot of the gain of the filter, that is the ratio between the amplitude of a sinusoidal input signal of period $P$ and the amplitude of the output sinusoidal signal when the time distance between two adjacent points is $\Delta t$. 
Figure 6: Plot of the phase differences. As can be seen there is no sign of the erratic behavior. The linear decrease of the amplitude is a clear sign of an error on the $P_{\text{orb}}$. An estimation of this correction with the usual methods is, if not impossible, extremely difficult.
Figure 7: Plot of the phase differences between the days 10 and 11 from the start of the observation. As can be seen there is only a sinusoidal modulation and there is no sign of the erratic behavior. The dotted line is the best-fit model described in Eq.2.5.
Figure 8: Pulse Phase delays as a function of time of XTE J1807–294 obtained after last iteration. The light curves are folded on a time interval of $P_{\text{orb}}$. It is now more apparent the erratic behavior of the source which characteristic times are at least of several dozen of $P_{\text{orb}}$. 
3.1 Abstract

We have performed a timing analysis of the 2003 outburst of the accreting X-ray millisecond pulsar XTE J1807–294 as observed by the Rossi X-Ray Timing Explorer. Using recently refined orbital parameters we report for the first time a precise estimate of the spin frequency and of the spin frequency derivative. The phase delays of the pulse profile show a strong erratic behavior superposed on what appears to be a global spin-up trend. The erratic behavior of the pulse phases is strongly related to rapid variations of the light curve, making it very difficult to fit these phase delays with a simple formula. As in previous cases, we therefore separately analyze the phase delays of the first harmonic and of the second harmonic of the spin frequency, finding that the phases of the second harmonic are far less affected by the erratic behavior. Under the hypothesis that the second harmonic pulse phase delays are a good tracer of the spin frequency evolution we give for the first time an estimate of the spin frequency derivative for this source. XTE J1807–294 shows a clear spin-up of $\dot{\nu} = 2.5(7) \times 10^{-14}$ Hz sec$^{-1}$ ($1 \sigma$ confidence level). The majority of the uncertainty in the value of the spin-up rate is due to the uncertainties in the source position on the sky. We discuss the effect of this systematic error on the spin frequency and its derivative.

3.2 Introduction

Binary systems in which one of the two stars is a neutron star (NS) are among the most powerful X-ray sources of our Galaxy. The emission of X-rays is due to the transfer of matter from the companion star and accretion onto the NS, and to the release of the immense gravitational energy during the infall or in the impact with the NS surface. A subcategory of such systems is referred to as low mass X-ray binaries (LMXBs), which are characterized by low NS surface magnetic fields ($< 10^9$ G) and by the low mass ($< 1 M_\odot$) of the companion star. In the so-called recycling scenario [see Bhattacharya e van den Heuvel, 1991, for a review], the millisecond radio pulsars are seen as the last
evolutionary step of LMXBs, where the torques due to the accretion of matter and angular momentum, coupled with the relatively weak magnetic fields, are able to spin up the NS to periods on the order of a millisecond. When the accretion phase terminates and the companion star stops transferring matter, the NS can switch on as a millisecond radio pulsar, although no example has been reported yet.

The recycling scenario received long-awaited confirmation only in 1998, with the discovery of the first millisecond X-ray pulsar in a transient LMXB: the first LMXB observed to show coherent pulsations, at a frequency of \(\sim 400\) Hz, was SAX J1808.4–3658 [Wijnands e van der Klis, 1998], in which the NS is orbiting its companion star with a period of \(\sim 2.5\) hr [Chakrabarty e Morgan, 1998]. Why millisecond X-ray pulsars were so elusive is an argument still debated in the literature. A possible reason is the relatively low magnetic fields of these sources, which therefore have a lessened capability to channel the accreting matter onto the polar caps, making the chance of seeing pulsed emission from an LMXB quite low [see, e.g., Vaughan et al., 1994], especially at high accretion rates. However, to date 10 LMXBs have been discovered to be accreting millisecond pulsars (see Wijnands, 2006 for a review of the first six discovered, for the last four, see Kaaret et al., 2006; Krimm et al., 2007; Casella et al., 2007; Altamirano et al., 2008), and all of them are in transient systems. They spend most of their time in a quiescent state, with very low luminosities (on the order of \(10^{31} - 10^{32}\) ergs s\(^{-1}\)) and rarely go into an X-ray outburst, with luminosities in the range \(10^{36} - 10^{37}\) ergs s\(^{-1}\). Although the recycling scenario seems to be confirmed by these discoveries, from timing analyses of accreting millisecond pulsars we now know that some of these sources exhibit spin-down while accreting [Galloway et al., 2002; Papitto et al., 2007]. This means that it is of fundamental importance to study the details of the mechanisms regulating the exchange of angular momentum between the NS and the accreting matter, which are far from being understood, and, chiefly, the role of the magnetic field in this exchange. The main way to do this is through the study of the pulse phase shifts and their relations with other physical observable parameters of the NS.

The pulse phase shifts are frequently affected by intrinsic long-term variations, fluctuations, or both (by which we mean an erratic behavior of the phase delays possibly caused by variations in the instantaneous accretion torques or movements of the accretion footprints on the NS surface; see di Salvo et al., 2007 for a review). Examples of this complex behavior in accreting millisecond pulsars have already been reported in the literature. Burderi et al. [2006] analyzed the 2002 outburst of the accreting millisecond pulsar SAX J1808.4–3658 and found a jump of 0.2 in the pulse phases of the first harmonic that is not present in the second-harmonic phases, which show much more regular behavior. This change corresponds to a change in the slope in the exponential decay of the X-ray light curve (see also Hartman et al. [2008] for a discussion of the complex phase behavior in other outbursts of SAX J1808.4–3658). Papitto et al. [2007] found that the second harmonic of XTE J1814–338 follows the first harmonic, giving approximately the same spin frequency derivative. A clear model that can explain this behavior is still lacking, but this observational evidence seems to suggest that perhaps the second harmonic has a more fundamental physical
meaning. For instance, it may be related to the emission of both polar caps, while the first harmonic may be dominated by the more intense but less stable polar cap. Another possible explanation comes from possible shape and/or size variations of the accretion footprints related to variations in the accretion rate. Romanova et al. [2003] found such a behavior in their numerical simulations.

In this paper, we report the results of a timing analysis performed on XTE J1807–294, making use of an improved orbital solution [Riggio et al., 2007]. As in the cases mentioned above, XTE J1807–294 shows erratic fluctuations in the phase delays of the first harmonic and a much more regular behavior of the phase delays derived from the second harmonic. Under the hypothesis that the second-harmonic pulse phase delays are a good tracer of the spin frequency evolution, we can derive a spin-up rate of $2.5(7) \times 10^{-14}$ Hz s$^{-1}$ ($1\sigma$ confidence level).

3.3 Observation and Data Analysis

The millisecond X-ray pulsar XTE J1807–294 was discovered by the Rossi X-Ray Timing Explorer (RXTE) on 2003 February 21 [Markwardt et al., 2003c]. The source was observed with the Proportional Counter Array (PCA; 2 - 60 keV energy range) and the High Energy X-Ray Timing Experiment (HEXTE; 20 - 200 keV) on board RXTE [Jahoda et al., 1996] during a long X-ray outburst that lasted from 2003 February 28 to June 22. XTE J1807–294 was also observed with other satellites such as XMM-Newton [Campana et al., 2003; Kirsch et al., 2004; Falanga et al., 2005b], Chandra [Markwardt et al., 2003a] and INTEGRAL [Campana et al., 2003]. No optical or radio counterpart has been reported to date. Linares et al. [2005] reported the presence of twin kilo hertz quasi periodic oscillations from an analysis of the RXTE observations.

Here we analyze all the archival RXTE observations of this source. In particular, we employ high time resolution data from the PCA. We use data collected in “GoodXenon” packing mode for the timing analysis, which permits maximum time and energy resolution (respectively, 1 µs and 256 energy channels). In order to improve the signal-to-noise ratio (S/N) we select photon events from the top layers of the proportional counter units (PCUs) in the energy range 3-13 keV [Galloway et al., 2002]. We have verified that this is the range where we indeed have the highest S/N. In fact, using the entire energy range the pulsations at days 104 and 106 after the beginning of the outburst are much less statistically significant.

Using the faxbary tool (DE 405 solar system ephemeris) we corrected the photon arrival times for the motion of the Earth-spacecraft system and referred them to Barycentric Dynamical Times at the solar system barycenter. We use the source position reported by Markwardt et al. [2003a] from a Chandra observation during the same outburst.

To obtain the X-ray light curve during the outburst, we used the PCA data collected in Standard-2 mode (256 energy channels and 16 s binned data) and corrected for the background using the faint-background model suitable for the source’s count rate [see Jahoda

faxbary is a tool of the HEASoft Software Packages. It can be found at: http://heasarc.nasa.gov/docs/software/lheasoft/
et al., 2006]. No energy selection was applied in this case since we are interested in a good tracer of the bolometric luminosity. We also did not apply any correction for dead time, since the maximum count rate was quite low (<100 counts s\(^{-1}\) per PCU, background included); in fact, the mean time between two events is at least 2 orders of magnitude higher than the expected dead time (10 µs) for this count rate [Jahoda et al., 2006]. We selected all the data using both internal Good Time Intervals and applying criteria regarding pointing offset, South Atlantic Anomaly (SAA) passage, electronic contamination, and Sun offset.\(^2\)

The resulting light curve is shown in Figure 9 (pentagons). The flux exhibits an exponential decay, with six evident flares superposed. To derive the characteristic time of the decay, we fitted the light curve with an exponential law. In order to remove the time intervals affected by X-ray flares we excluded from the fit all the points whose flux was greater than the best fit exponential model by at least 15%. This choice of threshold is arbitrary, but a different choice, such as 10% or 20%, includes or excludes very few other points. We repeated the exponential fit on the flare-subtracted light curve. In this last fit, the \(\chi^2\) was 23,096 for 214 degrees of freedom (dof), which is extremely high. Such a large \(\chi^2\) is due to deviations of the X-ray light curve from a pure exponential decay (see e.g. all the points after 100 days from the beginning of the outburst).

Although these deviations may be very small intrinsically, they can be large in comparison with the statistical error on a single point. However, in order to obtain a reliable estimate of the parameters of the fit, and in particular a reliable estimate of the errors, we need to obtain a reduced \(\chi^2\) of order unity. Therefore, we multiplied the errors on each point by a factor of 10. In this way we obtain a characteristic decay time of \(\tau = 17.50(25)\) days.

It should be noted that a constant term must be added to the model to obtain a good description of the light curve, although background subtraction was performed. This residual amounts to \(\sim 10.8(2)\) counts s\(^{-1}\) per PCU and may be due to a contaminating source in the PCA field of view. It is unlikely that it is due to quiescent emission, since the source was observed in quiescence by XMM-Newton and was not detected [Campana et al., 2005]. In either case, this residual flux does not affect the inferred decay time of the light curve or any other results of this paper.

In order to minimize the time delays induced by the orbital motion, we correct the photon arrival times according to the formula

\[
t_{\text{em}} \approx t_{\text{arr}} - A \left[ \sin (m(t_{\text{arr}}) + \omega) + \frac{\epsilon}{2} \sin (2m(t_{\text{arr}}) + \omega) - \frac{3\epsilon}{2} \sin (\omega) \right],
\]

[Deeter et al., 1981, see e.g.], where \(t_{\text{em}}\) is the photon emission time, \(t_{\text{arr}}\) is the photon arrival time, \(A\) is the projected semi-major axis in

\(^2\)According to the prescription given in http://heasarc.gsfc.nasa.gov/docs/xte/abc/screening.html we adopted as selection criteria the following: time since SAA greater than 30 minutes, elevation angle with respect the Earth greater than 10 degree, electron contamination lower than 0.1, and pointing offset lower than 0.25 degree.
light-seconds, \( m(t_{arr}) = 2\pi(t_{arr} - T^*)/P_{orb} \) is the mean anomaly with \( P_{orb} \) the orbital period and \( T^* \) is the time of ascending node passage, \( \omega \) is the periastron angle, and \( \epsilon \) the eccentricity. In order to remove completely from the pulse phase delays any effect due to the orbital motion, it is of fundamental importance to correct the arrival times of the events with very precise orbital parameters. To accomplish this, we used the orbital solution recently published by Riggio et al. [2007], who, using the total outburst time available (about 120 days), obtained a solution that is about 2 orders of magnitude more precise than previously reported orbital solutions.

We divided the whole observation into time intervals of length approximately equal to the orbital period and epoch-folded each of these data intervals with respect to the spin period reported in Table 3. In this way, we were able to significantly detect the X-ray pulsations up to day 106 from the beginning of the outburst, making this the longest time span over which a timing analysis of an accreting millisecond pulsar has been performed.

The pulse phase delays are obtained by fitting each pulse profile with two sinusoidal components (with periods fixed to multiples of 1 and 0.5 of the spin period, respectively), since the second harmonic was significantly detected in the folded light curve. In Figures 9 and 10, we show the pulse phase delays of the first and second harmonics, respectively. We have plotted only the pulse phase delays corresponding to the folded light curves for which the statistical significance for the presence of X-ray pulsations is greater than 3 \( \sigma \). Moreover, we consider the second harmonic significantly detected (and plot its phases) only when the ratio between the best fit amplitude of the second sinusoid and its error is greater than 3 (\( A/\delta A > 3 \)). For each phase point, we propagated the errors on the orbital parameters with the formulae derived in Burderi et al. [2007]. We note that the propagated errors in this case, for which the orbital parameters are known with great precision, come out to be much smaller than the statistical errors derived from the sinusoid fit.

As is evident from Figures 9 and 10, the phase delays of the first harmonic show a noisy behavior, with shifts of up to 0.3 in phase. This noise affecting the phase results is strongly anticorrelated with the source flux, as already noted for another source of this class [Papitto et al., 2007]. On the other hand, the phase delays derived from the second harmonic are much more regular, a behavior that is similar to that exhibited by SAX J1808.4–3658 [Burderi et al., 2006]. Although a few points (corresponding to rapid flares in the light curve) appear to be significantly below the general trend, the phase delays of the second harmonic clearly show a parabolic decrease, as is expected in the case of a spinning up of the NS.

### 3.4 Timing Results

Since the phase delays of the second harmonic are much less noisy than the phases derived from the first harmonic, and assuming that the

---

3 This is to minimize possible residuals due to uncertainties in the orbital parameters, since we expect these residuals to be periodic at the orbital period of the system.
pulse phase delays derived from the second harmonic are a good tracer of the spin frequency evolution, we decided to fit the second harmonic in order to find information about the spin frequency behavior. To fit the phase delays, we start from the simplest assumption of a constant spin frequency derivative. We hence fit the second-harmonic phase delays with the model

$$\phi(t) = \phi_0 - \Delta \nu (t - T_0) - \frac{\dot{\nu}}{2} (t - T_0)^2,$$

where $T_0$ is the date of the beginning of the observation, $\Delta \nu$ is a correction to the spin frequency, and $\dot{\nu}$ is the spin frequency derivative. Using all the data points we obtained a spin frequency derivative $\dot{\nu} = 2.05(28) \times 10^{-14}$ Hz s$^{-1}$ with a quite large $\chi^2$/dof = 1560.57/142. From a visual inspection of the phase residuals with respect to this model (see Fig. 10), we can see that the largest contribution to the $\chi^2$ is from a group of three points at MJD 52715.0 (about 14.5 days from the beginning of the outburst). These points (Fig. 10, triangles) correspond to the largest flare visible in the light curve and to a strong decrease of the phases of the first harmonic as well (cf. Fig. 9). We therefore believe that this is a phase shift induced by a rapid change of the X-ray flux similar to the phase shifts observed in the first harmonic. If we remove these three points from our data set ("case B") and perform the fit again, we obtain a frequency derivative $\dot{\nu} = 2.26(15) \times 10^{-14}$ Hz s$^{-1}$, perfectly compatible with the value previously found, demonstrating that the three points we have eliminated do not affect the spin frequency derivative obtained from the fit. In this case of course the statistical quality of the fit increases, giving $\chi^2$/dof = 452.4/139. However, this $\chi^2$ is still unacceptable; again, the postfit residuals indicate that the major contribution to the $\chi^2$ statistic is due to all the points corresponding to the X-ray flares. We therefore decided to remove all the points (Fig. 10, circles) that fall in time intervals during which the flux is higher by 15% with respect to the exponential best-fit function derived above. In this way, a total of 21 points were excluded from the fit ("case C"). With this last data set we obtain a value for the spin frequency derivative $\dot{\nu} = 2.46(15) \times 10^{-14}$ Hz s$^{-1}$ (again compatible with the results obtained with the complete data set) and $\chi^2$/dof = 257.6/121. In this case, a value of $\nu_0 = 190.623507018(6)$ Hz for the spin frequency at the beginning of the outburst is obtained.

We also tried to fit this (reduced) data set with a spin-up model that takes into account the decrease in the X-ray flux (supposed to trace the mass accretion rate) during the outburst (see Burderi et al., 2006 for a more detailed discussion). In principle, this correction should be important for this source given the particularly long duration of the outburst (about 120 days). Fitting the phase delays of the second harmonic with equation (1) of Burderi et al. [2006], in which we adopted an exponential decay time for the X-ray flux of 17.50(25) days, as derived from the X-ray light curve, we obtain a significant improvement in the fit, with $\chi^2$/dof = 225.5/121 ($\Delta \chi^2 = 32$ for the same number of degrees of freedom). In this case, we obtain a spin frequency derivative at the beginning of the outburst of $\dot{\nu}_0 = 1.25(7) \times 10^{-15}$ Hz s$^{-1}$, corresponding to a mass accretion rate at the beginning of the outburst of $M_0 = 4.03(23) \times 10^{-10}$ $M_\odot$ yr$^{-1}$, and a best fit spin frequency of
The Chandra observation of XTE J1807−294 was performed with the instrument HRC-S. As reported in http://asc.harvard.edu/cal/ASPECT/celmon/, the confidence levels are given at 68% (0″.4), 90% (0″.6) and 99% (0″.8).

\[ \gamma_0 = 190.623506939(7) \text{ Hz.} \]

In Figure 11 we report, for the last reduced data set, both parabolic and exponential best-fit models and the residuals from the exponential model (bottom).

Unfortunately, these results are affected by large systematic uncertainties due to the large uncertainty in the source coordinates (which is about \(0.4^\circ\) at 1σ confidence level) from the Chandra observation, which we next discuss in detail.

Uncertainties in the phase delays caused by uncertainties in the estimate of the source position on the sky will produce a sinusoidal oscillation at Earth’s orbital period. For observation times shorter than 1 yr, as is the case for most transient accreting millisecond pulsars, this can cause systematic errors in the determination of the NS spin period and its derivative, since a series expansion of a sinusoid contains both a linear and a quadratic term. In the case of XTE J1807−294, because of the low positional precision [Markwardt et al., 2003b] and the long time span over which the pulsations were visible (up to 106 days from the beginning of the outburst), we obtain, from the expression given in Burderi et al. [2007], the following systematic uncertainties on the spin frequency and the spin frequency derivative, respectively:

\[ \sigma_{\nu, \text{pos}} \sim 4.1 \times 10^{-8} \text{ Hz and } \sigma_{\nu, \text{pos}} \sim 0.8 \times 10^{-14} \text{ Hz s}^{-1}. \]

Since this error is of the same order of magnitude as our best-fit estimate of \(\dot{\nu}\), we need to evaluate these effects in a more careful manner.

Let us consider the expression for the phase delays induced by Earth’s motion for small displacements, \(\delta \lambda\) and \(\delta \beta\), in the position of the source in ecliptic coordinates \(\lambda\) and \(\beta\)

\[ \Delta \phi_{\text{pos}}(t) = \gamma_0 \gamma [\sin(M_0 + \epsilon) \cos \beta \delta \lambda - \cos(M_0 + \epsilon) \sin \beta \delta \beta] \]  

[see e.g. Lyne e Graham-Smith, 1990], where \(y = r_E/c\) is the distance of Earth from the solar system barycenter in light-seconds and \(M_0 = 2\pi(T_0 - T_y)/P_{\oplus} - \lambda\), with \(T_0\) the beginning of the observation, \(P_{\oplus}\) Earth’s orbital period, \(T_y\) the time of passage through the vernal point, and \(\epsilon = 2\pi(t - T_0)/P_{\oplus}\). As already calculated in Burderi et al. [2007], equation (3.3) can be rewritten as

\[ \Delta \phi_{\text{pos}} = \gamma_0 \gamma \sigma_\gamma \gamma \sin(M_0 + \epsilon - \theta^{*}) \]  

where \(\sigma_\gamma\) is the positional error circle, \(\theta^{*} = \arctan(\tan \beta \delta \beta/\delta \lambda)\), and \(\gamma = [(\cos \beta \delta \lambda)^2 + (\sin \beta \delta \beta)^2]^{1/2}/\sigma_\gamma\). We can safely impose \(\gamma = 1\) as an upper limit.

In order to take into account the effects of an incorrect source position, we fitted the reduced data set (case C) with a model that also takes into account the modulation caused by the incorrect source coordinates as given by equation (3.4)

\[ \phi(t) = \phi_0 - \Delta \nu (t - T_0) - \frac{\dot{\psi}}{2} (t - T_0)^2 + \Delta \phi_{\text{pos}}(t) \]  

We repeated the fit changing \(\sigma_\gamma\) and \(\theta^{*}\) in such a manner as to cover the Chandra error box up to the 90% confidence level, that is, a sky
region within an angular distance of 0.6″ from the reported source position. The obtained values of the spin frequency and mass accretion rate for each possible position of the source within the Chandra error box are shown in Figure 12. The values of $\dot{\nu}$, at the 1 σ confidence level, lie in the interval $(1.8 - 3.2) \times 10^{-14}$ Hz s$^{-1}$, while the best-fit value of the frequency derivative for the nominal source position is $2.46(15) \times 10^{-14}$ Hz s$^{-1}$. It is evident that the effect of the poor knowledge of the source position is much larger than the statistical error on the parabolic fit. Still, the spin-up behavior of the source remains significant even considering the large uncertainties caused by the positional uncertainties.

A similar discussion is required for the spin frequency. The best-fit value, for the nominal position, is $\nu = 190.623507018(4)$ Hz, while the variations of the linear term in the fit at different positions of the source inside the Chandra error box are in the range $\Delta \nu = \pm 4 \times 10^{-8}$ Hz, an order of magnitude greater than the single fit statistical error. Finally, the reduced $\chi^2$ for these fits varies within the range $(2.1 - 2.4)$.

To summarize, using the pulse phase delays derived from the second harmonic, we have inferred the spin frequency derivative of XTE J1807–294. Under the hypothesis of constant spin frequency derivative, we obtain a value $\dot{\nu} = 2.46(15) \times 10^{-14}$ Hz s$^{-1}$. Under the alternative hypothesis of an exponential decay of the accretion rate, we obtain a value for the spin frequency derivative at the beginning of the outburst of $\dot{\nu}_0 = 1.25(7) \times 10^{-13}$ Hz s$^{-1}$. These results do not include the systematic errors induced by the poorly constrained source position. Taking into account the errors on the source position obtained above, for the constant and exponential decay models, respectively, the values are $2.5(7) \times 10^{-14}$ Hz s$^{-1}$ and $1.25(33) \times 10^{-13}$ Hz s$^{-1}$.

### 3.5 Discussion and Conclusion

We have analyzed a long RXTE observation of the accreting millisecond pulsar XTE J1807–294 and reported the results of an accurate timing analysis on a time span of about 120 days, the longest outburst of an accreting millisecond pulsar for which a timing analysis has been performed to date. We find that the phase delays derived from the first harmonic show an erratic behavior around a global parabolic spin-up trend. This behavior is similar to that previously discovered in two accreting millisecond pulsars, SAX J1808.4–3658 [Burderi et al., 2006] and XTE J1814–338 [Papitto et al., 2007]. In the case of the 2002 outburst of SAX J1808.4–3658, the phase delays of the first harmonic show a shift by about 0.2 in phase at day 14 from the beginning of the outburst, when the X-ray flux abruptly changed the slope of the exponential decay. On the other hand, the phase delays of the second harmonic in SAX J1808.4–3658 showed no sign of the phase shift in the first harmonic and could be fitted by a spinning up during the first part of the outburst plus a barely significant spin-down at the end. In the case of XTE J1814–338, the fluctuations in the phase delays are visible both in the first harmonic and in the second harmonic, superposed on a global parabolic spin-down trend. Papitto et al. [2007] showed that the post-fit phase residuals are strongly anticorrelated with variations of the
X-ray light curve. These fluctuations were interpreted as being due to movements of the accretion footprints (or accretion column) induced by variations of the X-ray flux.

In the case of XTE J1807–294, the fluctuations in the phase delays mostly affect the first harmonic, which displays a trend that is very difficult to reproduce with a simple model. As in the case of XTE J1814–338, the postfit phase residuals are clearly anticorrelated with variations observed in the X-ray light curve; from Figure 9, we see that the phases decrease when the X-ray flux shows rapid increases. It is important to note that the anti-correlation visible between the postfit phase delays and the X-ray flux is independent of the spin-down or spin-up behavior of the source, since it is observed in XTE J1814–338, which shows spin-down, and in XTE J1807–294, which shows spin-up. The correlation between the phase delays and the X-ray flux affects the second harmonic only marginally. Indeed, there are a few points in the phase delays of the second harmonic that are significantly below the global trend observed in the phase delays, and they all correspond to flares in the X-ray light curve. Excluding these points marginally affects the values we obtain for the spin frequency and its derivative but produces a significant improvement in the $\chi^2$ of the fit.

We find that the phase delays of the second harmonic can be fitted by a parabolic spin-up model. We have also shown that the quality of the fit is much improved if a more physical model is used in which the spin-up rate decreases exponentially with time following the decrease of the X-ray flux (and hence of the inferred mass accretion rate). In fact, if the spin-up of the source is related to the mass accretion rate, then it should not be constant with time but, to first approximation, should decrease in proportion with the mass accretion rate onto the NS. For instance, assuming that the accretion of matter and angular momentum occurs at the corotation radius $R_{\text{co}}$, the relation between the spin frequency derivative and the mass accretion rate is, from the conservation of angular momentum, $\dot{\nu} = \dot{M}(GMR_{\text{co}})^{1/2}/2\pi I$, where $G$ is the gravitational constant, $M$ is the NS mass, and $I$ is the NS moment of inertia; this gives a lower limit on the mass accretion rate, since the specific angular momentum at the corotation radius is the maximum that can be transferred to the NS. In the case of XTE J1807–294, the duration of the observed outburst is particularly long (about 120 days), and the effect of a global decrease of the mass accretion rate during the outburst should be particularly relevant for this source. Indeed, in this case the fit we obtain using an exponentially decreasing spin-up rate is significantly better than that using a constant spin-up rate.

From the fit of the phase delays of the second harmonic of XTE J1807–294 with the model discussed above, we find a mass accretion rate at the beginning of the outburst of $4(1) \times 10^{-10} \, M_\odot \, \text{yr}^{-1}$. This mass accretion rate can be compared with the X-ray flux of the source at the beginning of the outburst, which was $2 \times 10^{-9} \, \text{ergs cm}^{-2} \, \text{s}^{-1}$ [Falanga et al., 2005b], and from which we derive an X-ray luminosity of $4.7 \times 10^{36} \, \text{ergs s}^{-1}$ and a distance to the source of 4.4(6) kpc.

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5 For this estimation we adopted the value of $I = 10^{45} \, \text{g cm}^2$, $M = 1.4 \, M_\odot$ and NS radius $R_{\text{NS}} = 10^6 \, \text{cm}$. 

Table 3: Orbital and Spin Parameters for XTE J1807-294.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA (J2000)</td>
<td>18h 06m 59.8s(^a)</td>
</tr>
<tr>
<td>Declination (J2000)</td>
<td>-29° 24' 30&quot;(^a)</td>
</tr>
<tr>
<td>Orbital period, (P_{\text{orb}}) (s)</td>
<td>2404.41665(40)(^b)</td>
</tr>
<tr>
<td>Projected semi-major axis, (a \sin i) (lt-ms)</td>
<td>4.819(4)(^b)</td>
</tr>
<tr>
<td>Ascending node passage, (T^*) (^a) (MJD)</td>
<td>52720.675603(6)(^b)</td>
</tr>
<tr>
<td>Eccentricity, (e)</td>
<td>&lt; 0.0036(^b)</td>
</tr>
<tr>
<td>Reference epoch, (T_0) (^c) (MJD)</td>
<td>52698.5</td>
</tr>
</tbody>
</table>

**Errors**

**Parabolic fit results**

- Spin frequency, \(\nu_0\) (Hz): 190.62350702(4)
- Spin frequency derivative, \(\dot{\nu}\) (Hz s\(^{-1}\)): \(2.5(7) \times 10^{-14}\)

**Exponential fit results**

- Spin frequency, \(\nu_0\) (Hz): 190.62350694(5)
- Spin frequency derivative, \(\dot{\nu}\) (Hz s\(^{-1}\)): \(1.25(33) \times 10^{-13}\)

On orbital parameters are intended to be at 1\(\sigma\) confidence level (c. l.), upper limits are given at 95\% c.l. Best fit spin parameters are derived in both hypothesis of a constant spin-up and flux dependent spin-up, and the uncertainties include systematics due to the uncertainties in the source position (see text).

\(^a\)Markwardt et al. [2003a].
\(^b\)Riggio et al. [2007].
\(^c\)This is the Epoch at which are referred the reported values of \(\nu\) and \(\dot{\nu}\).

Clearly, this is only a crude estimate of the distance on the basis of our timing results, and future independent estimates are needed in order to confirm or disprove our hypothesis.
Figure 9: Light curve of XTE J1807–294 during the 2003 outburst (pentagon) and phase delays of the first harmonic as a function of time (small dot). The dashed vertical lines indicate the times of six clearly visible flares of the X-ray flux superimposed to a global exponential decay. The dotted curve represent the exponential fit of the light curve, obtained after having previously excluded from the data the six flares. The dashed curve represent the parabolic best fit obtained fitting the second harmonic phase delays and considering the nominal source position. Strong fluctuations of the phase delays are apparent and are strongly anticorrelated to the flares present in the X-ray light curve.
Figure 10: Plot of XTE J1807–294 second harmonic pulse phase delays. The four curves represent the parabolic best fit for the nominal source position, respectively, using all the data points (case A), excluding the three point at MJD 52713.0 (case B, where the points excluded are identified by triangles), and excluding all the data points for which the flux exceeds the best-fit exponential decay for more than 15% (Case C, where the points excluded are identified by circles), and the best fit obtained using an exponentially decreasing mass accretion rate (see text). The exponential fit was performed on the data sub-set corresponding to case C.
Figure 11: second harmonic pulse phase delays together with the parabolic and exponential best fit (top panel), and residuals in units of $\sigma$ with respect to the exponential best fit model (bottom panel) considering only the subset of case C (see Fig. 10).
Figure 12: Diagrams of the best fit values of $\Delta \nu$ (left panel) and $\dot{M}$ (right panel) obtained fitting the first harmonic pulse phase delays with the expression $3.5$, as function of the parameters $\sigma_{\gamma}$ and $\theta^*$ (see text).
 Orbital Evolution of an Accreting Millisecond Pulsar: Witnessing the Banquet of a Hidden Black Widow?

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4.1 Abstract

We have performed a timing analysis of all the four X-ray outbursts from the accreting millisecond pulsar SAX J1808.4–3658 observed so far by the PCA on board RXTE. For each of the outbursts we derived the best-fit value of the time of ascending node passage. We find that these times follow a parabolic trend, which gives an orbital period derivative \( \dot{P}_{\text{orb}} = (3.40 \pm 0.18) \times 10^{-12} \text{s/s} \), and a refined estimate of the orbital period, \( P_{\text{orb}} = 7249.156499 \pm 1.8 \times 10^{-5} \text{s} \) (reference epoch \( T_0 = 50914.8099 \text{ MJD} \)). This derivative is positive, suggesting a degenerate or fully convective companion star, but is more than one order of magnitude higher than what is expected from secular evolution driven by angular momentum losses caused by gravitational radiation under the hypothesis of conservative mass transfer. Using simple considerations on the angular momentum of the system, we propose an explanation of this puzzling result assuming that during X-ray quiescence the source is ejecting matter (and angular momentum) from the inner Lagrangian point. We have also verified that this behavior is in agreement with a possible secular evolution of the system under the hypothesis of highly non-conservative mass transfer. In this case, we find stringent constraints on the masses of the two components of the binary system and its inclination. The proposed orbital evolution indicates that in this kind of sources the neutron star is capable to efficiently ablate the companion star, suggesting that this kind of objects are part of the population of the so-called black widow pulsars, still visible in X-rays during transient mass accretion episodes.

4.2 Introduction

SAX J1808.4–3658 is the first discovered among the ten known accreting millisecond pulsars (hereafter AMXPs), which are all transient X-ray sources, and is still the richest laboratory for timing studies of this class of objects. Although timing analysis have been now performed
on most of the sources of this sample with interesting results [see di Salvo et al., 2008b, for a review and references therein], SAX J1808.4–3658 is the only known AMXP for which several outbursts have been observed by the RXTE/PCA with high time resolution. In particular, the first outburst of this source was observed by the RXTE/PCA in April 1998, when coherent X-ray pulsations at ~2.5 ms and orbital period of ~2 hr [Wijnands e van der Klis, 1998; Chakrabarty e Morgan, 1998] were discovered. The source showed other X-ray outbursts in 2000 (when only the final part of the outburst could be observed, [Wijnands et al., 2001], in 2002 (when kHz QPOs and quasi-coherent oscillations during type-I X-ray bursts were discovered, Wijnands et al. 2003; Chakrabarty et al. 2003), and again in 2005, approximately every two years [see Wijnands, 2005, for a review].

Although widely observed, SAX J1808.4–3658 remains one of the most enigmatic sources among AMXPs, since timing analyses performed on this source have given puzzling results. Burderi et al. [2006], analysing the 2002 outburst, found that the pulse phases (namely the pulse arrival times) show evident shifts, probably caused by variations of the pulse profile shape. They noted that the phases derived from the second harmonic of the pulse profile were much more stable, and tentatively derived a spin frequency derivative from these data. To explain the relatively large frequency derivative a quite high mass accretion rate was required, about a factor of 2 higher than the extrapolated bolometric luminosity of the source during the same outburst. [Hartman et al., 2008, hereafter H08] performed a timing analysis of all the four outbursts of SAX J1808.4–3658 observed up to date by RXTE, finding again complex phase shifts in all of them. Their conclusion was that the large variations of the pulse shape do not allow to infer any spin frequency evolution during a single outburst, with typical upper limits of $|\dot{\nu}| \lesssim 2.5 \times 10^{-14}$ Hz/s (95% c.l.), which were derived excluding the first few days of the 2002 and 2005 outbursts and the large residuals at the 2002 mid-outburst. Interestingly, combining the results from all the analysed outbursts, H08 found a secular spin frequency derivative of $\dot{\nu} = (-5.6 \pm 2.0) \times 10^{-16}$ Hz/s, indicating a secular spin-down of the neutron star in this system. From this measure they found an upper limit of $1.5 \times 10^8$ Gauss to the neutron star magnetic field.

Papitto et al. [2005] performed a temporal analysis of the outbursts of SAX J1808.4–3658 that occurred in 1998, 2000, and 2002, which resulted in improved orbital parameters of the system. The large uncertainty caused by the relatively limited temporal baseline made it impossible to derive an estimate of the orbital period derivative. In this paper we use all the four outbursts of SAX J1808.4–3658 observed by RXTE/PCA, spanning a temporal baseline of more than 7 years, to derive an orbital period derivative, the first reported to date for an AMXP. The value we find with high statistical significance is surprising, $P_{\text{orb}} = (3.40 \pm 0.18) \times 10^{-12}$ s/s. This value for the orbital period derivative is compatible with the measure reported by H08, which, independently and simultaneously, have found the orbital period derivative of SAX J1808.4–3658. In § 3 we propose a simple explanation of this result arising from considerations on the conservation of the angular momentum of the system, which is consistent with a
(non-conservative) secular evolution of the system. In particular, we hypothesize that during quiescence SAX J1808.4–3658 experiences a highly non-conservative mass transfer, in which a great quantity of mass is lost from the system with a relatively high specific angular momentum.

### 4.3 Timing Analysis and Results

In this paper we analyse RXTE public archive data of SAX J1808.4–3658 taken during the April 1998 (Obs. ID P30411), the February 2000 (Obs. ID P40035), the October 2002 (Obs. ID P70080), and the June 2005 (Obs. ID P91056 and Obs. ID P91418) outbursts, respectively (see Wijnands 2005 and H08 for a detailed description of these observations). In particular, we analysed data from the PCA [Jahoda et al., 1996], which is composed of a set of five xenon proportional counters operating in the 2 – 60 keV energy range with a total effective area of 6000 cm². For the timing analysis, we used event mode data with 64 energy channels and a 122 µs temporal resolution. We considered only the events in the 3 – 13 keV energy range where the signal to noise ratio is the highest, but we checked that a different choice (considering for instance the whole RXTE/PCA energy range) does not change the results described below. The arrival times of all the events were referred to the solar system barycenter by using JPL DE-405 ephemerides along with spacecraft ephemerides. This task was performed with the faxbary tool, considering as the best estimate for the source coordinates the radio counterpart position, that has a 90% confidence error circle of 0.4 arcsec radius, which is compatible with that of the optical counterpart [Rupen et al., 2002; Giles et al., 1999].

For each of the outbursts we derived a precise orbital solution using standard techniques [see e.g. Burderi et al., 2007; Papitto et al., 2007, , and references therein]. In particular, we firstly corrected the arrival times of all the events with the orbital solution given by Papitto et al. [2005]. Then we looked for differential corrections to the adopted orbital parameters as described in the following. We epoch-folded time intervals with a duration of about 720 s each (1/10 of the orbital period) at the spin period of 2.49391975936 ms, and fitted each pulse profile obtained in this way with sinusoids in order to derive the pulse arrival times or pulse phases. The folding period was kept constant for all the outbursts. In fact, the determination of the pulse phases is insensitive to the exact value of the period chosen to fold the light curves providing that this value is not very far from the true spin period of the pulsar. Note that H08 have measured a secular derivative of the spin frequency in SAX J1808.4–3658, that is \( \dot{\nu} = (-5.6 \pm 2.0) \times 10^{-16} \) Hz/s. This is a quite small value, and they do not find evidence of variations of the spin frequency during a single outburst. In order to choose a value of the spin period as close as possible to the true one, we used the value above, that is in between the best-fit spin period reported by Chakrabarty & Morgan (1998) for the 1998 outburst and the best-fit spin period reported by Burderi et al. [2006] for the 2002 outburst. As in Burderi et al. [2006], we fitted each pulse profile with two sinusoids of fixed periods (the first, with period fixed to the spin period adopted
for the folding, corresponding to the fundamental, and the second, with period fixed to half the spin period, corresponding to the first overtone, respectively). In all cases the $\chi^2$/d.o.f. obtained from the fits of the pulse profiles was very close to (most of the times less than) 1. In order to improve the orbital solution we used the phase delays of the fundamental of the pulse profile which has the best statistics; the uncertainties on these phases were derived calculating 1-$\sigma$ statistical errors from the fit with two sinusoids.

We then looked for differential corrections to the adopted orbital parameters, which can be done by fitting the pulse phases as a function of time for each outburst. In general, any residual orbital modulation is superposed to a long-term variation of the phases, e.g. caused by a variation of the spin. However, as noted by Burderi et al. [2006] for the 2002 outburst, SAX J1808.4–3658 shows a very complex behavior of the pulse phases with time, with phase shifts, probably caused by variations of the pulse shape, that are difficult to model and to interpret. To avoid any fitting of this complex long-term variation of the phases, we preferred to restrict the fit of the differential corrections to the orbital parameters to intervals in which the long-term variation and/or shifts of phases are negligible. We therefore considered consecutive intervals with a duration of at least 4 orbital periods (depending on the statistics), and fitted the phases of each of these intervals with the formula for the differential corrections to the orbital parameters (see e.g. Deeter et al. 1981 and eq. (3) in Papitto et al. 2007). The selection of the intervals is somewhat arbitrary; we have verified, however, that the results do not change using a different choice. No significant corrections were found on the adopted values of the orbital period, $P_{\text{orb}}$, the projected semimajor axis of the neutron star (NS) orbit, $a_1 \sin i / c$, and the eccentricity of the orbit. In particular, for the eccentricity we find an upper limit of $4.6 \times 10^{-5}$ (95% c.l.) combining all the data.

On the other hand we found that the times of passage of the NS at the ascending node at the beginning of each outburst, $T_{\text{N}_0}^*$, were significantly different from their predicted values, $T_{\text{N}_0}^* + N P_{\text{orb}}$ where $T_{\text{N}_0}^*$ is the adopted time of ascending node passage at the beginning of the 1998 outburst and the integer $N$ is the exact number of orbital cycles elapsed between two different ascending node passages, i.e. $N$ is the integer part of $(T_{\text{N}_0}^* - T_{\text{N}_0}^*) / P_{\text{orb}}$ under the assumption that $|T_{\text{N}_0}^* - (T_{\text{N}_0}^* + N P_{\text{orb}})| < P_{\text{orb}}$ that we have also verified a posteriori. We therefore fixed the values of $P_{\text{orb}}$, $a_1 \sin i / c$, and the eccentricity, and derived the differential corrections, $\Delta T^*$, to the time of passage at the ascending node, obtaining a cluster of points for each outburst, which are plotted as a function of time in the inset of Figure 13. This has been done in order to check that (systematic) uncertainties on the arrival times of the pulses (such as phase shifts or other kind of noise) not already included in our estimated uncertainties for the pulse phases, did not significantly affect the determination of the orbital parameters. Since each of the points corresponding to an outburst may be considered like an independent estimate of the same quantity, if the errors on the phases were underestimated, we would expect that errors on the derived orbital parameters were underestimated. In this case the scattering of the points representing the time of passage at the ascending node derived for each of the considered intervals would be larger than
Figure 13: Differential correction, $\Delta T^*$, to the time of passage of the NS at the ascending node for each of the four outbursts analysed. In the inset we show the single measurements of $\Delta T^*$ obtained for each of the consecutive time intervals in which each outburst has been divided (see text). All the times are computed with respect to the beginning of the 1998 outburst, $T_0 = 50914.8099$ MJD.

The errors associated with each point. This indeed is not the case; in the inset of Fig. 13, we have shown the results obtained from each of these intervals, and the scattering of the points appears always comparable to the associated 1-$\sigma$ error. The largest scattering is observed for the outburst of 2000, which indeed is the point with the largest distance from the best fit parabola (see below). We think that this is caused by the worse statistics during this observation, which was taken at the end of the 2000 outburst.

We hence combined all the measurements corresponding to each outburst computing the error-weighted mean of the corresponding points, obtaining the four points shown in Figure 13. These points show a clear parabolic trend that we fitted to the formula:

$$\Delta T^* = \delta T_0^* + \delta P_{\text{orb}} \times N + (1/2)\dot{P}_{\text{orb}} P_{\text{orb}} \times N^2$$

In this way we found the best fit values $T_0^*$, $P_{\text{orb}} + \delta P_{\text{orb}}$ and $P_{\text{orb}}$ at $t = T_0$ shown in Table 4, with a $\chi^2 = 2.2$ (for 1 d.o.f.). This corresponds to a probability of 13% of obtaining a $\chi^2$ larger than the one we found. Our result is therefore acceptable (or, better, not rejectable), since the probability we obtained is above the conventionally accepted significance level of 5% (in this case, in fact, the discrepancy between the expected and observed values of $\chi^2$ is not significant since the two values are within less than 2 $\sigma$ from each other, see e.g. Bevington & Robinson 2003). We find a highly significant derivative of the orbital period, which indicates that the orbital period in this system is increasing at a rate of $(3.40 \pm 0.12) \times 10^{-12}$ s/s. Note that Ho8, simulta-
Table 4: Best fit orbital parameters for SAX J1808.4–3658.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0$ (days)</td>
<td>50914.8784320 (11)</td>
</tr>
<tr>
<td>$P_{\text{orb}}$ (s)</td>
<td>7249.156499 (18)</td>
</tr>
<tr>
<td>$\dot{P}_{\text{orb}}$ (s/s)</td>
<td>$3.40(18) \times 10^{-12}$</td>
</tr>
<tr>
<td>$a_1 \sin i$ (lt-ms)</td>
<td>62.809 (1) \textsuperscript{a}</td>
</tr>
<tr>
<td>$e$</td>
<td>$&lt; 4.6 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\chi^2$/dof</td>
<td>0.98/1</td>
</tr>
</tbody>
</table>

The reference time at which the orbital period, $P_{\text{orb}}$ and its derivative, $\dot{P}_{\text{orb}}$, are referred to is the beginning of the 1998 outburst, that is $T_0 = 50914.8099$ MJD. Numbers in parentheses are the uncertainties in the last significant digits at 90\% c.l. Upper limits are at 95\% c.l. Uncertainties are calculated conservatively increasing the errors on the fitted points in order to reach a $\chi^2$/dof $\simeq 1$, as described in the text.

\textsuperscript{a} The value of $a_1 \sin i$ and its 1-$\sigma$ error are from Chakrabarty & Morgan 1998.

neously and independently, found a very similar result for the orbital period derivative in SAX J1808.4–3658, $\dot{P}_{\text{orb}} = (3.5 \pm 0.2) \times 10^{-12}$ s/s, compatible with the result reported in this paper. The only difference is in the quoted error. H08 state that this difference is possibly due to an underestimate of the phase uncertainties reported in this paper, as testified by a worse reduced $\chi^2$. We just note that we do not have any evidence that the phase uncertainties we derive are underestimated. Indeed our reduced $\chi^2$ is worse than the one obtained by H08, that is $\chi^2 = 1.01$ for 1 dof), but it is still statistically acceptable. However, in the hypothesis that our errors are slightly underestimated, we have increased by a factor of 1.5 all the errors on the phases in order to obtain a $\chi^2$ as close as possible to 1. In this way we obtain a $\chi^2$ of 0.98 for 1 d.o.f., and we have re-evaluated the errors on the orbital parameters, finding that these errors increase by a factor of 1.5. This means that our "conservative" estimate of the orbital period derivative is $(3.40 \pm 0.18) \times 10^{-12}$ s/s (90\% c.l.). As a final check we have fitted with the same formula the points shown in the inset of Figure 13, obtaining, as expected, the same results.

Note that the orbital period of SAX J1808.4–3658 is now known with a precision of 1 over $10^5$, that is an improvement of one order of magnitude respect to the previous estimate by Papitto et al. [2005] and two orders of magnitude with respect to Chakrabarty e Morgan [1998]. On the other hand, the derivative of the orbital period indicates that the orbital period in this system is increasing at the quite large rate of $(3.40 \pm 0.18) \times 10^{-12}$ s/s, that is at least an order of magnitude higher than what is predicted by a conservative mass transfer driven by Gravitational Radiation (GR, see below). In the next section we discuss a possible explanation for this surprising result.
We have performed a precise timing analysis of all the X-ray outbursts of the AMXP SAX J1808.4–3658 observed to date by the RXTE/PCA, covering more than 7 years in time. We divided each outburst in several intervals and found, for each interval, differential corrections to previously published orbital parameters. The obtained times of passage of the NS at the ascending node were significantly different in different outbursts. We fitted these times with a parabolic function of time finding an improved orbital solution valid over a time span of more than 7 years. This solution includes a highly significant derivative of the orbital period, \( \dot{P}_{\text{orb}} = (3.40 \pm 0.18) \times 10^{-12} \text{ s/s} \). This value, found simultaneously and independently by Ho8, is the first measure of the orbital period derivative for an AMXP. However, this orbital period derivative is quite unexpected, since it is more than one order of magnitude higher than what is expected from conservative mass transfer driven by GR.

Orbital evolution calculations show that the orbital period derivative caused by conservative mass transfer induced by emission of GR is given by:

\[
\dot{P}_{\text{orb}} = -1.4 \times 10^{-13} \ m_1 \ m_{2,0.1} \ m^{-1/3} p_{2h}^{-5/3} \left( \frac{[n-1/3]/[n+5/3-2q]}{2} \right) \text{ss}^{-1}
\]

(derived from Verbunt 1993; see also Rappaport et al. 1987), where \( m_1 \) and \( m \) are, respectively, the mass of the primary, \( M_1 \), and the total mass, \( M_1 + M_2 \), in units of solar masses, \( m_{2,0.1} \) is the mass of the secondary in units of 0.1 \( M_\odot \), \( p_{2h} \) is the orbital period in units of 2 h, \( q = m_2/m_1 \) is the mass ratio and where \( n \) is the index of the mass-radius relation of the secondary \( R_2 \propto M_2^n \). Therefore a positive orbital period derivative certainly indicates a mass-radius index \( n < 1/3 \), and therefore, most probably, a degenerate or fully convective companion star [see e.g. King, 1988]. However, the \( \dot{P}_{\text{orb}} \) we measure is an order of magnitude higher than what is expected from GR.

### 4.4.1 Conservation of angular momentum

In order to explain the quite unexpected value measured for the orbital period derivative, we start from the equation of the angular momentum of the system, which must be verified instantaneously. The orbital angular momentum of the system can be written as: \( I_{\text{orb}} = \left[ G a / (M_1 + M_2) \right]^{1/2} M_1 M_2 \), where \( G \) is the Gravitational constant, and \( a \) is the orbital separation. We can differentiate this expression in order to find the variation of the orbital angular momentum of the system caused by mass transfer. We indicate with \( -M_2 \) the mass transferred by the secondary, which can be accreted onto the neutron star (conservative mass transfer) or can be lost from the system (non-conservative mass transfer). We can therefore write \( M_1 = -\beta M_2 \), where \( \beta \) is the fraction of the transferred mass that is accreting onto the neutron star, while \( 1 - \beta \) is the fraction of the transferred mass that is lost from the system. The specific angular momentum, \( l_{ej} \), with
which the transferred mass is lost from the system can be written in units of the specific angular momentum of the secondary, that is: 

\[ \alpha = 1 \frac{L_\text{X-ray}}{2\pi a^2 M_2^2} = 1 \frac{L_\text{orb}}{(M_1 + M_2)^2/2\pi a^2 M_1^2}, \]

where \( L_\text{X-ray} \) is the X-ray luminosity from the source, that is \( \sim 34 \times 10^{-16} \text{ ergs/s} \). Since SAX J1808.4−3658 accretes for about 30 days every two years, we have estimated an order of magnitude for the averaged X-ray luminosity from the source, that is \( L_\text{X} \sim 4 \times 10^{34} \text{ ergs/s} \), which corresponds to \( 3 \frac{M_2}{M_1} = 6.6 \times 10^{-16} \text{ s}^{-1} \). It is easy to see that the measured \( P_\text{orb}/P_\text{orb,} 4.7 \times 10^{-16} \text{ s}^{-1} \), is at least 70 times higher than the value predicted in the conservative mass transfer case, hence excluding this scenario.

Assuming a totally non-conservative mass transfer, \( \beta = 0 \), we find that \( g(1, q, \alpha) = 1 - q \sim 1 \), where we have used the information that for SAX J1808.4−3658 the mass function gives \( q \geq 4 \times 10^{-2} \) for \( M_1 = 1.4 M_\odot \) (Chakrabarty & Morgan 1998), and can be therefore neglected. Hence, for conservative mass transfer the conservation of angular momentum gives: 

\[ \frac{P_\text{orb}}{P_\text{orb}} \leq 3 \left( -\frac{M_2}{M_1} g(\beta, q, \alpha) \right). \]

(4.4)

Assuming a conservative mass transfer, \( \beta = 1 \), it is easy to see that \( g(1, q, \alpha) = 1 - q \sim 1 \), where we have used the information that for SAX J1808.4−3658 the mass function gives \( q \geq 4 \times 10^{-2} \) for \( M_1 = 1.4 M_\odot \) (Chakrabarty & Morgan 1998), and can be therefore neglected. Hence, for conservative mass transfer the conservation of angular momentum gives: 

\[ \frac{P_\text{orb}}{P_\text{orb}} \leq 3 \left( -\frac{M_2}{M_1} g(\beta, q, \alpha) \right). \]

(4.4)

Assuming a totally non-conservative mass transfer, \( \beta = 0 \), we find that \( g(0, q, \alpha) = 1 - \alpha + 2q/3)/(1 + q) \sim 1 - \alpha \), implying that 

\[ \frac{P_\text{orb}}{P_\text{orb}} \leq 3(1 - \alpha)(-\frac{M_2}{M_1}). \]

Since the first term is positive we find that \( \alpha < 1 \) and the specific angular momentum with which matter is expelled from the system must be less than the specific angular momentum of the secondary. For matter leaving the system with the specific angular momentum of the primary we have \( \alpha = q^2 - 0 \). In this case we find, therefore, the same result of the conservative case where no angular momentum losses from the system occur. This is due to the fact that the specific angular momentum of the primary is so small that there is no difference with respect to the conservative case. Since the specific angular momentum of the mass lost must be in between the specific angular momentum of the primary and that of the secondary, a reasonable hypothesis is that matter leaves the system with the specific angular momentum of the inner Lagrangian point. In this case 

\[ \alpha = |1 - 0.462(1 + q)^{2/3} q^{1/3}|^2 \approx 0.7, \]

where we have used for the Roche Lobe radius the approximation given by Paczyński (1971). We therefore find 

\[ \frac{P_\text{orb}}{P_\text{orb}} \leq (-\frac{M_2}{M_1}). \]

Using the measured value of the quantity \( P_\text{orb}/P_\text{orb} = 4.7 \times 10^{-16} \text{ s}^{-1} \), we find that to explain this result in a totally non-conservative scenario the mass transfer rate must
be: \((- M_2) = M_{\odot} \geq 8.3 \times 10^{-10} M_{\odot} \text{ yr}^{-1}\). We can therefore explain the measured derivative of the orbital period of the system assuming that the system is expelling matter at a quite large rate, that may be as high as \(\sim 10^{-9} M_{\odot} \text{ yr}^{-1}\), and this is found just assuming the conservation of the angular momentum of the system, and independently of the secular evolution adopted.

### 4.4.2 Possible secular evolution of the system

In order to verify whether this result is just a transient peculiar behavior of the system due to unknown causes or it is instead compatible with a secular evolution, we have solved the secular evolution equations of the system using the assumptions described in the following. i) Angular momentum losses are due to GR and are given by: 

\[
\dot{J}/J = -32G^3 M_1 M_2 (M_1 + M_2)/(5c^5 a^4),
\]

where \(c\) is the speed of light, and \(J = M_1 M_2 [G a/(M_1 + M_2)]^{1/2}\) is the binary angular momentum [Landau e Lifshitz, 1975; Verbunt, 1993, see e.g.]. ii) For the secondary we have adopted a mass-radius relation \(R_2 \propto M_2^n\). iii) We have imposed that the radius of the secondary follows the evolution of the secondary Roche Lobe radius: \(R_{12}/R_{12} = R_2/R_2\), where for the radius of the secondary Roche Lobe we have adopted the Paczyński (1971) approximation: 

\[
R_{12} = 2/3^{4/3} [q/(1 + q)]^{1/3} a
\]

that is valid for small mass ratios, \(q = M_2/M_1 \lesssim 0.8\). In these hypotheses we derived a simple expression for the orbital period derivative and the mass transfer rate in the extreme cases of totally conservative and totally non-conservative mass transfer [see e.g. Verbunt, 1993; van Teeseling e King, 1998; King et al., 2003, 2005]:

\[
\dot{P}_{\text{orb}} = -1.38 \times 10^{-12} \left[\frac{n-1/3}{n-1/3+2g}\right] m_1^{8/3} q(1+q)^{-1/3} P_{2h}^{-5/3} \text{ s/s}
\]  

(4.5)

\[
\dot{M} = -M_2 = 4.03 \times 10^{-9} \left[\frac{1}{n-1/3+2g}\right] m_1^{8/3} q^2 (1+q)^{-1/3} P_{2h}^{-8/3} M_{\odot}/\text{yr}
\]  

(4.6)

where \(g = 1 - q\) for totally conservative mass transfer (in this case it is easy to see that eq. 4.5 gives the same \(\dot{P}_{\text{orb}}\) given by eq. 4.4, as expected), and \(g = 1 - (\alpha + q/3)/(1 + q)\) for totally non-conservative mass transfer.

Comparing the measured orbital period derivative with eq. 4.5 (assuming that the orbital period derivative we measure reflects the secular evolution of the system), we note that, in order to have \(\dot{P}_{\text{orb}} > 0\) we have to assume an index \(n < 1/3\). In the case of SAX J1808.4–3658 the secondary mass is \(m_2 \leq 0.14\) at 95% c.l. (Chakrabarty & Morgan 1998), and therefore the mass ratio \(q \leq 0.1\) implies that for the totally conservative mass transfer case (\(g = 1 - q\)), the \(\dot{P}_{\text{orb}}\) expected from GR must be of the order of \(10^{-13} \text{ s/s}\), not compatible with what we measured for SAX J1808.4–3658. In other words, if we assume a conservative
mass transfer for the system (that means that the mass transferred during outbursts is completely accreted by the NS, and during quiescence no or negligible mass is accreted or lost from the system), we find that it is impossible to explain the observed orbital period derivative with a secular evolution driven by GR.

On the other hand, if we assume that during X-ray quiescence the companion star is still overflowing its Roche Lobe but the transferred mass is not accreted onto the NS and is instead ejected from the system, we find a good agreement between the measured and expected orbital period derivative assuming that the matter leaves the system with the specific angular momentum at the inner Lagrangian point, \( \alpha = \left[ 1 - (2/3)^{4/3}\right] q^{1/3}(1 + q)^{2/3} \). Adopting the measured value \( \dot{P}_{\text{orb}} = 3.4 \times 10^{-12} \text{ s/s} \) and the other parameters of SAX J1808.4–3658, eq. 4.5 translates into a relation between \( m_1 \), \( m_2 \) and the mass-radius index \( n \); this is plotted in Figure 14 (top panel) for different values of \( n \) going from 0 to \( n = -1/3 \). The constraint on \( m_1 \) vs. \( m_2 \) imposed by the mass function of the system is also plotted (the shadowed region in the figure) and indicates that the most probable value for \( n \) is \(-1/3\), which in turn indicates a degenerate or, most probably, a fully convective companion star. In fact, in a system with orbital period less than 3 h, where the mass of the Roche-Lobe filling companion is below \( 0.2 - 0.3 \ M_\odot \), the companion star becomes fully convective with a mass-radius hydrostatic equilibrium equation \( R \propto M^{-1/3} \) (e.g. King 1988; Verbunt 1993). Also, for reasonable minimum, average and maximum values of the NS mass, 1.1, 1.56, and 2.2 \( \text{M}_\odot \), respectively, we obtain the following values for the secondary mass: 0.053, 0.088, and 0.137 \( \text{M}_\odot \), and the following values for the inclination of the system: 44°, 32°, and 26°, respectively.

Assuming therefore \( n = -1/3 \) we have plotted in Figure 14 (bottom panel) the corresponding non-conservative mass transfer rate as a function of \( m_1 \). We find that for \( m_1 = 1.5 \) the mass transfer rate must be of the order of \( 10^{-9} \text{ M}_\odot/\text{yr} \), much higher than what is expected in a conservative GR driven mass transfer case. Note that this high \( M \) might explain the spin period evolution reported by Burderi et al. (2006; see, however, H08 who could not detect a spin period derivative during the outbursts). Actually, during the X-ray outbursts, the mass transfer is conservative since the transferred matter is accreted onto the NS. However, the accretion phase duty cycle, about 40 days / 2 years = 5%, is so small that the totally non-conservative scenario proposed above is a good approximation.

4.4.3 Is SAX J1808.4–3658 a ‘hidden’ black widow pulsar?

If the hypothesis of a highly non-conservative mass transfer in SAX J1808.4–3658 is correct, the question to answer is why is accretion inhibited during X-ray quiescence while the companion star is transferring mass at a high rate? We propose that the answer has to be found in the radiation pressure of the magneto-dipole rotator emission, with a mechanism that is similar to what is proposed to explain the behavior of the so-called black widow pulsars (see e.g. Tavani 1991a; King et al. 2003, 2005; see also Burderi et al. 2001). Indeed, the possibility that
Figure 14: **Top:** Companion star mass vs. NS mass in the hypothesis of totally non conservative mass transfer (with matter leaving the system with the specific angular momentum at the inner Lagrangian point) and assuming the \( P_{\text{orb}} \) measured for SAX J1808.4–3658. Different curves correspond to different values of the mass-radius index \( n \) of the secondary. Horizontal lines indicate the limits for the secondary star mass corresponding to reasonable limits for the NS mass and to \( n = -1/3 \).

**Bottom:** Mass rate outflowing the secondary Roche Lobe in the hypothesis of totally non conservative mass transfer (as above) and assuming \( n = -1/3 \).
the magneto-dipole emission is active in SAX J1808.4−3658 during X-ray quiescence has been invoked by Burderi et al. (2003, see similar results in Campana et al. 2004) to explain the optical counterpart of the source, which is observed to be over-luminous during quiescence [Homer et al., 2001]. In this scenario, the optical luminosity in quiescence is explained by the spin-down luminosity of the magneto-dipole rotator (with a magnetic field of $(1-5) \times 10^8$ Gauss) which is reprocessed by the companion star and/or a remnant accretion disc. Interestingly, similar evidence of a strongly irradiated companion star during quiescence has been found also for IGR J00291+5934, the fastest among the known AMXPs [D’Avanzo et al., 2007].

In other words, a temporary reduction of the mass-accretion rate onto the neutron star (note that the so-called disc Instability Model, DIM – see e.g. Dubus et al. 2001 – usually invoked to explain the transient behavior of these sources, may play a role in triggering or quenching the X-ray outbursts in SAX J1808.4−3658) may cause the switch on of the emission of the magneto-dipole rotator, and, in some cases, even if the mass transfer rate has not changed, the accretion of matter onto the NS can be inhibited because the radiation pressure from the radio pulsar may be capable of ejecting out of the system most of the matter overflowing from the companion [see e.g. Burderi et al., 2001, and references therein]. This phase has been termed “radio–ejection”. One of the strongest predictions of this model is the presence, during the radio-ejection phase, of a strong wind of matter emanating from the system: the mass overflowing from the companion swept away by the radiation pressure of the pulsar. Indeed, the existence of ‘hidden’ millisecond pulsars, whose radio emission is completely blocked by material engulfing the system that is continuously replenished by the mass outflow driven by companion irradiation, has already been predicted by Tavani [1991a].

A beautiful confirmation of this model was provided by the discovery of PSR J1740–5340, an eclipsing millisecond radio pulsar, with a spin period of 3.65 ms, located in the globular cluster NGC 6397 [D’Amico et al., 2001]. It has the longest orbital period ($P_{\text{orb}} \approx 32.5$ hrs) among the 10 eclipsing pulsars detected up to now. The peculiarity of this source is that the companion is a slightly evolved turnoff star still overflowing its Roche lobe. This is demonstrated by the presence of matter around the system that causes long lasting and sometimes irregular radio eclipses, and by the shape of the optical light curve, which is well modeled assuming a Roche-lobe deformation of the mass losing component [Ferraro et al., 2001]. An evolutionary scenario for this system has been proposed by Burderi et al. [2002], who provided convincing evidence that PSR J1740–5340 is an example of a system in the radio-ejection phase, by modeling the evolution of the possible binary system progenitor. In other words, PSR J1740–5340 can be considered a ‘star-vaporizing pulsar’ of type II in the terminology used by Tavani [1991a].

We believe that the behavior of SAX J1808.4−3658 is very similar to the one of PSR J1740–5340, the main differences being the orbital period, which is ~ 32 h in the case of PSR J1740–5340 and ~ 2 h in the case of SAX J1808.4−3658, and the mass transfer rate from the companion, which has been estimated to be ~ $10^{-10}$ $M_{\odot}$ yr$^{-1}$ for PSR...
J1740–5340 and one order of magnitude higher for SAX J1808.4–3658. Both these factors will increase the local Dispersion Measure (DM) to the source in the case of SAX J1808.4–3658, and hence will predict a much higher free-free absorption in the case of SAX J1808.4–3658. This is in agreement with the fact that, although widely searched, no radio pulsations have been detected from SAX J1808.4–3658 up to date [Burgay et al., 2003]. In other words, SAX J1808.4–3658 can be considered a ‘hidden’ millisecond pulsar, or a ‘star-vaporizing pulsar’ of type III in the terminology used by Tavani [1991a].

A similar highly non conservative mass transfer, triggered by irradiation of the secondary and/or of an accretion disc by the primary (according to the model of Tavani 1991b, has been proposed to explain the large orbital period derivative observed in the ultra-compact Low Mass X-ray Binary X 1916–053, composed of a neutron star and a semidegenerated white dwarf, exhibiting periodic X-ray dips. In this case, \( \dot{P}_{\text{orb}}/P_{\text{orb}} \approx 5.1 \times 10^{-15} \text{ s}^{-1} \), which implies a mass transfer rate of \( \sim 10^{-8} \text{ M}_\odot \text{ yr}^{-1} \), with 60 – 90% of the companion mass loss outflowing from the system [Hu et al., 2008].

As predicted by several authors [Chakrabarty e Morgan, 1998; King et al., 2005, e.g.], and in agreement with our interpretation of the orbital period derivative in SAX J1808.4–3658 as due to a highly non-conservative mass transfer, we propose therefore that SAX J1808.4–3658, and other similar systems, belong to the population of the so-called black widow pulsars (or are evolving towards black widow pulsars); these are millisecond radio pulsars thought to ablate the companion and likely able to produce large mass outflows. When (or if) the pressure of the outflowing matter becomes sufficiently high to temporarily overcome the radiation pressure of the magneto-dipole rotator, the source experiences a transient mass accretion episode, resulting in an X-ray outburst. Indeed, SAX J1808.4–3658 and the other known AMXPs are all transient systems (accreting just for a very short fraction of the time), with small values of the mass function (implying small minimum mass for the secondary) and short orbital periods (less than a few hours).\(^1\)

Although in some of these black widow radio pulsars (variable) radio eclipses have been observed, clearly demonstrating the presence of matter around the system, a direct proof of severe mass losses from these system has never been found to date. The orbital evolution of SAX J1808.4–3658 indicates that this X-ray transient millisecond pulsar indeed may expel mass from the system for most of the time with just short episodes of accretion observed as X-ray outbursts. We therefore propose that SAX J1808.4–3658 (and perhaps most AMXPs) is indeed a black widow still eating the companion star.

However, some black widow pulsars have shown quite complex derivatives of the orbital period. In particular, the prototype of this class, PSR B1957+20 (a type-I star-vaporizing eclipsing millisecond pulsar), shows a large first derivative of the orbital period (almost an order

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\(^1\) Among the presently known AMXPs, the newly discovered intermittent pulsar SAX J1748.9–2021 (Altamirano et al. 2008) in the globular cluster NGC 6440 is an exception since this pulsar shows mass function and orbital period higher than the other Galactic AMXP. However, it is worth noting that this pulsar belongs to a globular cluster in which capture in the dense cluster environment may have played a role.
of magnitude higher than the one of SAX J1808.4–3658, and also a second orbital period derivative, indicating a quasi-cyclic orbital period variation [Arzoumanian et al., 1994]. A similar behavior has been observed in another binary millisecond pulsar, PSR J2051–0827, which shows a third derivative of the orbital period and a variation of $a \sin i$, indicating that the companion is underfilling its Roche lobe by $\sim 50\%$ [Doroshenko et al., 2001]. In these cases, the complex orbital period variation has been ascribed to gravitational quadrupole coupling (i.e. a variable quadrupole moment of the companion which is due to a cyclic spin-up and spin-down of the star rotation). In this scenario, the companion star must be partially non-degenerate, convective and magnetically active, so that the wind of the companion star will result in a strong torque which tends to slow down the star, making the companion star rotation deviating from synchronous rotation ($\Omega_C = \frac{2\pi}{P_{\text{orb}}}$, with $f < 1$).

In the case of SAX J1808.4–3658, the present data do not allow to find a second derivative of the orbital period, and therefore it is not clear if the orbital period derivative will change sign. However, we note that for a (type-I) black widow (radio) pulsar the companion star may be no more strongly orbitally locked; hence deviations from co-rotation may be possible, and this may reflect in strange (cyclic) changes of the orbital period of the system. However, in the case of SAX J1808.4–3658, that is an accreting neutron star, the companion must be completely orbitally locked (since the secondary star still fills its Roche lobe during X-ray outbursts and cannot detach in a time scale of a few years) and therefore the most probable way to change the orbital period of the system is a change of the averaged specific angular momentum, which can be obtained transferring mass from the secondary to the neutron star or expelling mass from the system with appropriate specific angular momentum, less than the specific angular momentum of the secondary.

In conclusion, we propose that we are witnessing the behavior of a ‘hidden’ black widow, eating its companion during X-ray outbursts and ablating it during quiescence; the next X-ray outburst of SAX J1808.4–3658 will be of fundamental importance to test or disprove this scenario. Moreover, the analogy of SAX J1808.4–3658 with a black widow should also be tested observationally; observing the source at other wavelength may give important information. For instance, it is already known that SAX J1808.4–3658 shows transient radio emissions; this was observed for the first time at the end of the 1998 outburst, approximately 1 day after the onset of a rapid decline in the X-ray flux, by Gaensler et al. [1999] and was attributed to an ejection of material from the system. The possible presence of an Hα bow shock nebula (like the one observed in PSR B1957+20) may be difficult to test in this case, given the crowding around the source in the optical band [Campana et al., 2004, see], while it may be important to look for an IR excess which may be caused by excess of matter around the system.
TIMING OF THE 2008 OUTBURST OF SAX J1808.4-3658 WITH XMM-NEWTON: A STABLE ORBITAL PERIOD DERIVATIVE OVER THE LAST TEN YEARS

5.1 ABSTRACT

We report on a timing analysis performed on a 62-ks long XMM-Newton observation of the accreting millisecond pulsar SAX J1808.4-3658 during the latest X-ray outburst which started on September 21st 2008. By connecting the time of arrivals of the pulses observed during the XMM-Newton observation we derived the best fit orbital solution and a best fit value of the spin period for the 2008 outburst. Comparing these new set of orbital parameters, and in particular the value of the time of ascending node passage, with the orbital parameters derived for the previous four X-ray outbursts of SAX J1808.4-3658 observed by the PCA on board the Rossi-XTE, we find an updated value of the orbital period derivative, which results to be $\dot{P}_{\text{orb}} = (3.89 \pm 0.15) \times 10^{-12}$ s/s. This new value of the orbital period derivative is in agreement with the value previously reported by Hartman et al. [2008] and di Salvo et al. [2008a], demonstrating that the orbital period derivative in this source has remained stable over the last ten years. This strongly supports a highly non-conservative mass transfer in this system, where the accreted mass (as derived from the X-ray luminosity during outbursts) accounts for a mere 1% of the mass lost by the companion.

5.2 INTRODUCTION

Accreting Millisecond Pulsars (AMSP) are generally interpreted as the evolutionary link between the Low Mass X-ray binaries (LMXB) and rotation powered Millisecond Radio Pulsars. According to the recycling scenario the latter are in fact formed after a phase of mass accretion from a low mass companion star, which ultimately spins the neutron star up to ms periods [see e.g. Bhattacharya e van den Heuvel, 1991]. SAX J1808.4-3658 was the first AMSP discovered [Wijnands e van der Klis, 1998]. In ten years the population of AMSP has grown to ten sources, all residing in close and transient binaries, but SAX J1808.4-3658 can still be considered the cornerstone of its class as it has repeatedly gone in outburst almost every two years after its first detection, making it the most observationally rich source of its class. Its timing and orbital properties have been extensively studied; while the timing behavior of the spin frequency is still debated because of the presence of timing noise in the phase delays (see Burderi et al. [2006], B06 hereinafter; Hartman et al. [2008], Ho8 hereinafter) there is a good agreement as regards the orbital parameters of this source, which are now known with extreme precision (Ho8; di Salvo et al. [2008a], dS08 hereinafter).
In particular a timing of the past four outbursts of SAX J1808.4–3658 observed with the Proportional Counter Array (PCA) on board the Rossi-XTE satellite (spanning more than 7 years from April 1998 to October 2005) has allowed to derive the orbital period of this source with a relative uncertainty $\Delta P_{\text{orb}}/P_{\text{orb}}$ of 2.5 and $1.9 \times 10^{-9}$ (dS08 and H08, respectively). More interesting, the orbital period of the source shows a significant derivative which indicates that it is increasing at a large as well as puzzling rate of $\dot{P}_{\text{orb}} = (3.4 \pm 0.2) \times 10^{-12}$ s/s (dS08 and H08, respectively). The positive derivative of the orbital period indicates a degenerate or fully convective companion star (which are characterized by a mass-radius relation of the secondary $R_2 \propto M_2^n$ with $n < 1/3$). However, the value of the orbital period derivative is at least one order of magnitude higher than what is expected for conservative mass transfer driven by Gravitational Radiation (GR).

In order to confirm this trend of the orbital, and to further improve the orbital solution for this interesting system, we have analysed a 62 ks XMM-Newton Target of Opportunity (ToO) observation of this source, performed during the latest outburst in October 2008. The spectral analysis of these data is discussed in a companion paper [Papitto et al., 2008b], while in this paper we report on the temporal analysis of this XMM-Newton observation.

5.3 Observation and Data Reduction

SAX J1808.4–3658 was found in outburst on 2008 September 21 by RXTE, and since then it was object of an intensive observational campaign. A preliminary analysis of the 2–10 keV Swift XRT publicly available light curve shows that the outburst had its maximum around September 24. XMM-Newton observed SAX J1808.4–3658 as a ToO observation for 62 ks on 2008 October 1 (start time MJD 54739.99517), roughly one week after the assumed outburst peak. The EPIC-pn camera operated in timing mode, to prevent photon pile-up and to allow an analysis of the coherent and aperiodic timing behaviour of the source. The same observing mode was used for EPIC/MOS2 CCDs, while EPIC/MOS1 was operated in small window mode, and the RGS in spectroscopy mode. A bright external flare in the background is present during the first 2 ks of the observation, so that we excluded this time interval from our analysis.

The high source flux saturated the XMM-Newton telemetry rate, so that the XMM-Newton Science Operations Centre decided to switch off one of the MOS camera (MOS2) in order to allocate more band to the EPIC-pn instrument, roughly 35 ks after the beginning of the observation. The EPIC-pn power spectrum showed no noticeable difference before and after the MOS-2 turning off, and the whole EPIC–pn data set is therefore considered for our analysis. The MOS cameras have a much lower effective area with respect to the pn, resulting in a much lower statistics while operating in the same energy range. Similar arguments hold for the RGS data. We therefore used only data collected by the pn camera for our temporal analysis.

Data were extracted and reduced using SAS v.8.0.0; we produced a calibrated EPIC–pn event list through the epproc pipeline. The arrival
times of all the events were referred to the solar system barycenter by using barycen tool in SAS. We considered as the best estimate for the source coordinates the position of the radio counterpart, which has an uncertainty of 0.4 arcsec (90% c.l., Rupen et al. 2002) and is compatible with the optical counterpart [Giles et al., 1999].

In Figure 15 we show the SAX J1808.4–3658 light curve of the XMM-Newton observation, using pn data and a bin time of 100 s; the count rate appears to rise by about 10% from the beginning to the end of the observation.

5.4 Timing Analysis and Results

We derived a precise orbital solution by analysing the XMM-Newton ToO observation of 2008 October 1 together with RXTE public archive data of SAX J1808.4–3658 taken during the April 1998 (Obs. ID P30411), the February 2000 (Obs. ID P40035), the October 2002 (Obs. ID P70080), and the June 2005 (Obs. ID P91056 and Obs. ID P91418) outbursts, respectively [see Wijnands, 2005, and H08 for a detailed description of these observations]. For each of the outbursts we derived a precise orbital solution using standard iterative techniques [see e.g. Burderi et al., 2007; Papitto et al., 2007, and references therein].

In particular, in the first iteration, we corrected the arrival times of all the events with the orbital solution given by dS08, namely $T^*|_{0'}$, $P_{\text{orb}}|_{0}$, and $a_1 \sin i/c|_{0}$, where the notation $|_{\text{rm}i}$ refers to the step of the iteration: the starting values for the first iteration ($i = 1$) are

The actual $T^*|_{0}$ used in this work is the $T^*|_{0}$ of dS08 decremented by $P_{\text{orb}}|_{0}$ in order to have the time of ascending node passage just before the beginning of RXTE data of the 1998 outburst which occurred at $T_0 = 50914.8099$ MJD.
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(i - 1 = 0) in this notation. Then we looked for differential corrections to the adopted orbital parameters as described in the following. We epoch-folded time intervals with a duration of about 700 s each (~ 1/10 of the orbital period) at the spin period of 2.49391975936 ms (as in dS08), and fitted each pulse profile obtained in this way with sinusoids in order to derive the pulse arrival times or pulse phases. The folding period was kept constant for all the outbursts. As already observed in dS08, we note that the determination of the pulse phases is insensitive to the exact value of the period chosen to fold the light curves providing that this value is not very far from the true spin period of the pulsar.

In order to choose a value of the spin period as close as possible to the true one, we used the value above, that is in between the best-fit spin period reported by Chakrabarty e Morgan [1998] for the 1998 outburst and the best-fit spin period reported by B06 for the 2002 outburst. As in B06, we fitted each pulse profile with two sinusoids of fixed periods (the first, with period fixed to the spin period adopted for the folding, corresponding to the fundamental, and the second, with period fixed to half the spin period, corresponding to the first overtone, respectively). In all cases the $\chi^2$/d.o.f. obtained from the fits of the pulse profiles was very close to (most of the times less than) 1. In order to improve the orbital solution we used the phase delays of the fundamental of the pulse profile which has the best statistics; the uncertainties on these phases were derived calculating 1-σ statistical errors from the fit with two sinusoids. We did not considered phases obtained from pulse profiles in which the amplitude of the fundamental was less than 3 σ.

We then looked for differential corrections to the adopted orbital parameters, which can be done by fitting the pulse phases as a function of time for each outburst. In general, any residual orbital modulation is superposed to a long-term variation of the phases, e.g. caused by a variation of the spin. However, as noted by B06 for the 2002 outburst, SAX J1808.4--3658 shows a very complex behavior of the pulse phases with time, with phase shifts, probably caused by variations of the pulse shape, that are difficult to interpret. To model this complex behavior of the pulse phases with time, we fitted the phases of each of these intervals with the formula for the differential corrections to the orbital parameters [see e.g. Deeter et al., 1981] plus a polynomial (up to 8th degree, depending on the irregular behavior of the pulse phases). This technique is complementary to the technique adopted in dS08, where we preferred to restrict the fit of the differential corrections to the orbital parameters to intervals in which the long-term variation and/or shifts of phases were negligible and considered consecutive intervals with a duration of at least 4 orbital periods (depending on the statistics), fitting the phases of each of these intervals only with the formula for the differential corrections of the orbital parameters. We have verified that the two techniques give orbital corrections compatible within their errors. No significant corrections were found on the adopted value of the orbital period, $P_{\text{orb}}$. We therefore decided to keep this parameter fixed, since we can improve its measure with the same technique adopted in dS08. For each of the five outburst we therefore find similar corrections for the projected semimajor axis of the neutron star orbit, $\delta a_1 \sin i/c_m_{\text{1}}$ (with $m = 1998, 2000, 2002, 2005$, and 2008), and upper limits on the eccentricity. We hence combined
all the measurements corresponding to each outburst computing the error-weighted mean of the corrections to the projected semimajor axis \( < \delta a_1 \sin i/c|_1 > \) and computed the first step corrected semimajor axis as \( a_1 \sin i/c|_1 = a_1 \sin i/c|_0 + < \delta a_1 \sin i/c|_1 > \), and the error-weighted mean on the upper limit on the eccentricity. On the other hand we found that the correction \( \delta T_m^*|_1 \) to the predicted times of passage of the NS at the ascending node at the beginning of each outburst, \( T_{m \text{ predicted}}|_1 = T^*|_0 + NP_{\text{orb}|_0}^2 \), were significantly different between each others. We then plotted these “Times of Ascending Node Passage Delays”, namely \( \delta T_m^*|_1 \) vs time. These points show a clear parabolic trend that we fitted to the formula:

\[
\delta T_m^*|_i = \delta T^*|_i + \delta P_{\text{orb}|_i} = N + \frac{1}{2}P_{\text{orb}|_i}T_{\text{orb}|_i}(i - 1) \times N^2
\]

where \( \delta T^*|_i \), \( \delta P_{\text{orb}|_i} \), and \( P_{\text{orb}|_i} \) are the fit parameters (in particular \( P_{\text{orb}|_i} \) is the orbital period derivative obtained at the \( i \)th iteration). In this way, after the first iteration, we found the best fit value for the orbital period \( P_{\text{orb}|_1} = P_{\text{orb}|_0} + \delta P_{\text{orb}|_1} \) at \( t = T^*|_0 \).

In the second iteration we adopted \( a_1 \sin i/c|_1 \), \( P_{\text{orb}|_2} \), and the five values (derived from the first iteration fits) for the “Times of Ascending Node Passage” at the beginning of each outburst, \( T_m^*|_1 = \delta T_m^*|_1 + T_{m \text{ predicted}}|_1 \), to correct, for each outburst, the arrival times of all the events. We repeated the procedure described above and found a new upper limit on the eccentricity, \( a_1 \sin i/c|_2 \), \( P_{\text{orb}|_2} \), and five values of \( T_m^*|_2 \). We then iterated this method again to find that that the upper limit on the eccentricity, and the corrections \( \delta a_1 \sin i/c|_2 \) were all compatible with zero within the computed errors.

Again we found that all the \( T_m^*|_3 \)s were significantly different from \( T_{m \text{ predicted}}|_3 \) (which were computed by assuming the constant \( P_{\text{orb}|_2} \) and we fitted the resulting \( \delta T^*|_3 \)s with a parabola, as described above, deriving the final values for \( \delta T^*|_3 \), \( \delta P_{\text{orb}|_3} \) (which was compatible with zero within the computed error, as expected), and \( P_{\text{orb}|_3} \), with a reduced \( \chi^2 = 1.55 \) (for 2 d.o.f.). This corresponds to a probability of 21% of obtaining a \( \chi^2 \) larger than the one we found. Our result is therefore acceptable (or, better, not rejectable), since the probability we obtained is well above the conventionally accepted significance level of 5% [see e.g. Bevington e Robinson, 2003]. In particular we found a highly significant derivative of the orbital period, which indicates that the orbital period in this system is increasing at a rate of \( P_{\text{orb}} = 3.89 \pm 0.15 \times 10^{-12} \) s/s. The best fit values for the orbital parameters are shown in Table 5. With these new ephemeris we corrected the events of the XMM-Newton ToO observation of 2008 October 1 and fitted the 700

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2 The integer \( N \) is the exact number of orbital cycles elapsed between two different ascending node passages, i.e. \( N \) is the closest integer to \( |T_{m|1} - T^*|_0|/P_{\text{orb}|0} \) under the assumption that \( |T_{m|1} - T_{m \text{ predicted}}|_1| \ll P_{\text{orb}|1} \) that we have verified a posteriori.

3 We observed that fitting any data set as short as few days for each outburst gave values for the \( T_{m|1} \) that were consistent within the errors with the \( T_{m|1} \)s obtained using the whole data set for each outburst (tens of days). Thus the computed \( T_{m|1} \)s were insensitive to the time coordinate adopted within the duration of the whole observation. Therefore, in order to compute the correct errors on the derived parameters, we considered the errors induced by the uncertainties in the time coordinate (which we assumed to be half of the duration of each data set) following the standard procedure for the propagation of these errors [see e.g. Bevington e Robinson, 2003].
s folded phase delays of the fundamental with a staright line in order to obtain the correction to the spin frequency adopted for the folding. The fit was acceptable with a reduced $\chi^2 = 1.16$ (for 71 d.o.f.). The best fit spin frequency of the 2008 outburts is also reported in Table 5. In Figure 16 we plot the phase delays obtained with the solution of dS08 neglecting the orbital period derivative. The sinusoidal oscillation is mainly due to the $\sim 35$ s delay of the ascending node passage with respect to the one predicted with a constant orbital period. In Figure 17 we plot the pulse profile obtained from the XMM-Newton/pn data. The frequency adopted for the folding is the best fit spin frequency of the 2008 outburst. In Figure 18 we plotted the final $\Delta T^*_m|_{3s}$ vs time, together with the best fit parabola.

5.5 Discussion

In this paper we present a timing analysis of the XMM-Newton observation performed during the last outburst of the AMSP SAX J1808.4–3658, together with a re-analysis of the past four outbursts from this source observed by RXTE/PCA; in this last case we have performed a more conservative treatment of the errors in the determination of the times of periastron passage which takes into account systematic uncertainties caused by phase shifts on short time scales. Putting together the five measures we have of the time of periastron passages for the last 5 outbursts from this source, we find that these point are again perfectly fitted by a parabola; from this fitting we find a derivative of the orbital period, $\dot{P}_{\text{orb}} = (3.89 \pm 0.15) \times 10^{-12}$ s/s (90% c.l.), which is in agreement with previous measures (H08; dS08; see also ? which report a similar value using RXTE data taken during the 2008 outburst). The conclusion from these measurements is that the orbital period derivative in this system has been stable over the last ten years.

As derived in dS08, angular momentum conservation and the third Kepler’s law give:

$$\frac{\dot{P}_{\text{orb}}}{P_{\text{orb}}} = 3 \left[ \frac{j}{I_{\text{orb}}} - \frac{M_2}{M_1} g(\beta, q, \alpha) \right], \quad (5.2)$$

Table 5: Best fit orbital solution for SAX J1808.4–3658 derived from the analysis of the five outbursts observed from 1998 to 2008.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^*$</td>
<td>MJD</td>
<td>50914.79452952(85)</td>
</tr>
<tr>
<td>$P_{\text{orb}}$ at $t = T^*$</td>
<td>s</td>
<td>7249.156444(23)</td>
</tr>
<tr>
<td>$\dot{P}_{\text{orb}}$</td>
<td>$10^{-12}$ s/s</td>
<td>3.89(15)</td>
</tr>
<tr>
<td>$a_1 \sin i/c$</td>
<td>s</td>
<td>0.0628106(20)</td>
</tr>
<tr>
<td>$e$</td>
<td></td>
<td>$&lt; 1.2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\nu$ at $t = T^*$</td>
<td>Hz</td>
<td>400.9752084532(17)</td>
</tr>
</tbody>
</table>

Note: Errors are at 1σ c.l. on the last 2 digits. The upper limit on the eccentricity is at 95% c.l.
Figure 16: Phase delays (in µs) vs. time during the XMM-Newton observation after the first iteration. To show for clarity the orbital correction against an horizontal line, the frequency adopted for the folding is the best fit spin frequency of the 2008 outburst. The orbital modulation is clearly visible (top panel) and is well fitted by our orbital solution. In the middle panel we show the time residuals after our first improvement of the orbital solution, and in the bottom panel the time residuals with respect to our final orbital solution. Note that in this case more points are visible in the residuals. This is caused by the fact that improving the orbital solution results in a higher statistical significance of each point, making acceptable points that were rejected with our previous solution.
Figure 17: 64-bin pulse profile obtained from folding the XMM-Newton/pn data (after correcting the arrival times of all the events with our best orbital solution) at the best fit spin frequency of the 2008 outburst. For sake of clarity two spin periods are plotted. The dashed lines on top the data represent the single components (fundamental and first harmonic, respectively) obtained from the fit of the pulse profile.

Figure 18: Differential correction to the time of passage of the neutron star at the ascending node for each of the five outbursts analysed (top panel) and residuals with respect to the best fit parabola (bottom panel).
where $1/\text{orb}$ represents possible losses of angular momentum from the system (e.g. caused by GR), $M_1$, $M_2$, $M_1$ and $M_2$, $M_1$ are the masses of the neutron star, the companion and their time derivatives, respectively, $\beta$ is the fraction of the mass lost by the companion that is accreting onto the neutron star ($M_1 = -\beta M_2$), $q = M_2/M_1$, $\alpha$ is the specific angular momentum of the mass lost by the system written in units of the specific angular momentum of the companion ($\alpha = [1 - 0.462(1 + q)^{2/3} q^{1/3}]^2 \approx 0.7$ for matter that leaves the system with the specific angular momentum of the inner Lagrangian point), and $g(\beta, q, \alpha) = 1 - \beta q - (1 - \beta)(\alpha + q/3)/(1 + q)$. Considering the GR angular momentum losses, the equation above can be written (in units appropriate for SAX J1808.4–3658) as:

$$P_{-12} = 0.138\ m_{1}^{5/3} q_{0.1}^{1}(1 + 0.1 q_{0.1})^{-1/3} p_{\text{orb}}^{-5/3} + 6.840\ m_{1}^{-1} q_{0.1}^{-1} p_{\text{zh}} g(\beta, q, \alpha) m_{-9}$$

(5.3)

where $P_{-12}$ is $P_{\text{orb}}$ in units of $10^{-12}$ s, $m_1 = M_1/M_{\odot}$, $q_{0.1} = q/0.1$, $p_{\text{orb}} = P_{\text{orb}}/2h$, and $m_{-9} = -M_2/(10^{-9} M_{\odot}/\text{yr})$.

The value of the orbital period derivative clearly indicate that the binary system is expanding at a rate that is more than one order of magnitude higher than the prediction of conservative mass transfer driven by GR (Ho8; dS08). In fact, the angular momentum losses caused by GR have the effect to reduce the binary orbital period. The transfer of mass from the secondary to the neutron star has indeed the effect to expand the system (since the secondary star is lighter than the neutron star), e.g. Verbunt (1993). For conservative mass transfer $g = (1 + 0.1 q_{0.1})$. Inserting our derived value $P_{-12} = 3.89$, the equation above can be solved for $m_{-9}$ vs $m_2 = q m_1$ (for $m_1 = 1.1, 1.56, 2.2$, as in dS08). The corresponding curves (which are quite insensitive to the $m_1$ adopted) are shown in Figure 19 and labelled with $\beta = 1$. In the same figure the horizontal line at $m_{-9} = 0.017$ is the time averaged mass accretion rate derived from the outburst observations and computed as follows. We choose as a prototype of the lightcurve of SAX J1808.4–3658 that of the 2002 outburst, which has probably been the more intense up to date. As described in B06, the lightcurve shows an exponential decay with a timescale $\tau = 9.27$ days up to day 14 after the beginning of the outburst, after this day the exponential decay steepens with $\tau \approx 3$ days. From the value obtained for the spin frequency derivative, B06 derived $m_{-9} = 1.8$ which is a factor 2 higher than the bolometric luminosity inferred by SAX J1808.4–3658 RXTE spectra. This gives the total amount of mass accreted per outburst $\Delta m = 3.6 \times 10^{-11} M_{\odot}$. Considering the five outburst which occurred in 3826 days we derived $m_{-9} = 0.003$ (which could even be an overestimate by a factor 2 of the actual mass transfer rate). The crossing point of these curves at $m_2 \approx 0.003$ is the necessary solution imposed by the conservation of angular momentum for this system. Adopting a quite standard mass-radius relation for H-rich (X ~ 0.6) white dwarfs, namely Hydrogen Brown Dwarfs $R_2/R_{\odot} = 1.169 \times 10^{-2} m_2^{-1/3}$ (which e.g. gives the correct prediction for the radius of Jupiter) we can impose the obvious constraint $R_2 \leq R_{\text{RL}}$, where $R_2$ is the radius of the companion and $R_{\text{RL}} = 1.2 \times 10^{10} m_1^{1/3} q_{0.1}^{1/3} P_{\text{zh}}^{2/3}$ cm is the Roche Lobe.
Figure 19: Mass loss rate of the companion of SAX J1808.4–3658 in conservative ($\beta = 1$) and highly non-conservative ($\beta = 0.01$) scenarios. The grey area is forbidden because the radius of the companion should be greater than its Roche Lobe.
radius of the companion. The grey area in Figure 19 indicates the region excluded by the violation of this constraint and shows that a conservative solution is impossible. On the other hand, in line with dS08, we propose a highly non-conservative scenario in which the mass loss rate from the companion is quite stable around the value derived in B06, namely $m_{\sim 9} = 1.8$, with accretion episodes lasting few tens of days, separated by quiescent phases in which the same rate of mass is ejected by the system. In this hypothesis, the same calculation as above gives $\beta = 9.5 \times 10^{-3} \sim 0.01$. The corresponding lines (which are computed for $m_1 = 1.1, 1.56, 2.2$) are shown in Figure 19 and labelled with $\beta = 0.01$. The crossing point between the non-conservative lines and the horizontal line for $m_{\sim 9} = 1.8$ is outside the forbidden area and demonstrates that our proposed scenario is viable.

Ho8 note that the orbital parameters of SAX J1808.4–3658 are very similar to those of the so-called black widow millisecond pulsars, which show large and variable $P_{\text{orb}}$. In particular, the prototype of this class, PSR B1957+20, observed over a timespan of about five years, shows a large positive first derivative of the orbital period (almost an order of magnitude higher than the one of SAX J1808.4–3658), with a complex short-term behavior (which sometimes results in large negative orbital period derivative) and also a second orbital period derivative, indicating a quasi-cyclic orbital period variation (Arzoumanian, Fruchter, & Taylor 1994). A similarly complex behavior, with a third derivative of the orbital period and a variation of $a \sin i$ has been observed in another binary millisecond pulsar, PSR J2051–0827 (Doroshenko et al. 2001). However, the timing of SAX J1808.4–3658, which is monitored since its discovery as a millisecond pulsar in 1998, does not show any evidence of these complex behaviors, and instead appears to be particularly stable in the years. (LIMITI SULLA DERIVATA SECONDA DELL’ORBITAL PERIÓD DERIVATIVE: 1.13e-20 +/- 1.2e-20 CHECK!)

As discussed by dS08, the difference between SAX J1808.4–3658 and these typical black widow pulsars may be that in the case of SAX J1808.4–3658, that is an accreting neutron star, the companion must be completely orbitally locked (since the secondary star still fills its Roche lobe during X-ray outbursts and cannot detach in a time scale of a few years), while for the black widow millisecond pulsars cited above there is evidence that the companion is underfilling its Roche lobe (by a factor of $\sim 50\%$, in the case of PSR J2051–0827, Doroshenko et al. 2001). In other words, for black widow (radio) pulsar the companion star may be no more strongly orbitally locked, and hence through complex tidal coupling it acts as a source or a sink of angular momentum that can be exchanged with the orbital one, which ultimately causes the complex evolution of the orbital period observed in these systems. On the other hand, in the case of SAX J1808.4–3658, to explain the large orbital period derivative, we are therefore left with just one possibility; a large mass transfer rate is needed to explain the observed orbital period derivative. However, this is not observed to accrete onto the neutron star, and hence must be expelled from the system. These results thus strength the hypothesis of dS08, which proposed that SAX J1808.4–3658 belongs to the population of the so-called ‘hidden’ millisecond pulsars, whose radio emission is completely blocked by material engulfing the system that is continuously replenished by the mass out-


flow driven by companion irradiation (e.g. Tavani 1991). In the case of SAX J1808.4–3658 irradiation of the companion star is probably due to the power emitted by the magneto-dipole rotator, which may also explain why, although the companion star is transferring mass at a high rate, this mass does not accrete onto the neutron star (see dS08 for a more detailed discussion); if this is the case, ~ 99% of the transferred mass in this system is not directly observable.
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