We see only what we know.

An introduction to inverse problems with applications

Inverse solutions are key problems in science. They form the basis of our way to "see" the world surrounding us. Whenever we try to learn something about physical laws, the internal structure of the Earth, the nature of the Universe, how quarks are distributed inside the hadrons, we collect data and try to extract the required information from these data. This is the actual solution of inverse problems. The observed data are determined by the physical laws and the structure of the investigated objects (Earth, Universe, hadrons). Predicting observed data for given sources within a given system is referred to as the forward problem. The inverse problem consists of using the actual result of some measurements to infer the values of the parameters that characterize the system.

Probably I should say that inverse solutions are key problems in our everyday life, not only in science; even vision is an inverse problem: from 2-D stimuli we try to reconstruct 3-D objects; from the visual experience we obtain information about physical space. In this case (as in all other inverse problems, as I will try to discuss) the role of our a priori knowledge about objects is of primary importance. In this sense, "we see only what we know".

Inversion of real data is complicated by the fact that real data are invariably contaminated by noise and are acquired at a limited number of observation points. Moreover, mathematical models are complicated and yet at the same time also simplifications of the true physical phenomena. As a result, the solutions are ambiguous and error-prone. The principal issues arising in inverse problems are about the existence, uniqueness and stability of the solution.

In mathematics, we have a classical definition of ill-posed problem: a problem is ill-posed if the solution does not exist, is not unique or if it is not stable. A way to tackle this kind of problems is based on the application of different type of so-called "regularization" algorithms, which allow automatic selection of the proper solution by means of a priori information. The central point of this short course is the application of this kind of algorithms. These algorithms use a priori information about the target (e.g. the existence of sharp boundaries between media with different elastic waves velocities, in the case of seismic tomography) to reduce the ambiguity and the instability of the solution.

So, in short, during the course, I will try to discuss: a few examples of forward and inverse problems (in particular in geophysics); uniqueness and instability in the solution of inverse problems; ill-posed problems and methods to solve them; regularization; principles of discrete inverse theory; linear and nonlinear inversion strategies; introduction to gradient-type methods.
The attendees will be involved through practical examples and computational exercises.